The physical interest in Kd and $\bar{p}d$ atoms

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Exotic deuterium and helium are discussed. The S, P and D levels of $\bar{p}$ and K^- atoms are calculated. Absorptive, subthreshold $\bar{p}N$ amplitudes are extracted from experimental data and compared to model calculations. The existence of a quasi-bound state in the $\bar{p}N$ system is indicated. In the K^- atoms some effects of $\Sigma(1385)$ resonance are evaluated.

1 Introduction

The lightest hadronic atoms offer a chance to test hadron-nucleon scattering amplitudes at and just below the thresholds. This energy region is of special interest when quasi-bound states exist in the hadron-nucleon system. The two cases of current interest, $\bar{p}N$ and KN belong to this category.

Several experiments with antiprotonic hydrogen, deuterium and helium atoms have been performed [1, 2, 3, 4]. These require an analysis which is different from the standard optical model description of heavier atoms. In particular, the best fit optical model parameters bear no clear relation to the low energy $\bar{p}N$ scattering parameters. However, there is a good chance to find such a relation in the light atoms and we attempt it in this paper.

In the KN system, two strange baryonic resonances $\Lambda(1405)$ and $\Sigma(1385)$ are well known. However, their coupling to the K meson nucleon state is not fully understood. One point of interest in this field is the old question of the nature of $\Lambda(1405)$ state interpreted as an elementary particle [5], a KN quasi-bound state [6] or a mixture of both. The $1S$ Kd level shift is expected to shed some light on this question. Our particular interest in this system is motivated by the recent discovery of the nuclear $KNNN$ [7] state and possible role of the $\Sigma(1385)$, [8]. We look for the effect of this resonance upon the "upper" widths of K atoms.

Two results are presented in this paper:

• The atomic level shifts are related to the zero energy scattering amplitudes of the orbital hadron on the atomic nucleus. In light $\bar{p}$ atoms the threshold parameters (lengths, volumes) may be extracted from the $\bar{p}d$, $\bar{p}^3$He, $\bar{p}^4$He data. Next, a simple formula is used which expresses these parameters in terms of the $\bar{p}N$ amplitudes averaged over some subthreshold energy region. The latter: lengths - a and volumes- b are taken as free parameters and are extracted from the data. Such a program can be performed for S, P states in $\bar{p}d$ and P, D states in $\bar{p}$He. In these nuclei, the nucleons are bound differently and thus different $\bar{p}N$ energies are involved. Thus, one obtains $a(E), b(E)$ which indicate distinct energy dependence in the subthreshold region. A unique resolution is obtained in the case of Im $a(E)$ and Im $b(E)$. In this case additional data from the $\bar{p}$ stopped in d and He chambers [9, 10] allow to disclose the isospin content of the absorptive amplitudes. Finally, Im $a(E), b(E)$ are compared to the results of an updated Paris potential model [16]. A good understanding of the data is obtained. It indicates a $\bar{p}N$ quasi-bound state in a P wave.

• In the K^- atoms we calculate the chances to reach the 1S state in deuteron and estimate the uncertainties involved in the related levels. A point of our special interest is the contribution from the $\Sigma(1385)$ to the upper level widths in $K-d$ and K-He systems.

2 The relation of level shifts to scattering amplitudes

Experiments which detect the X-rays emitted from hadronic atoms provide atomic levels shifted and widened by nuclear interactions. For a given n-th state of angular momentum L the full energy E_{nL} is shifted from the electromagnetic level ϵ_{nL} by a complex level shifts $\Delta E_{nL} = \delta E_{nL} - i\Gamma_{nL}/2 = E_{nL} - \epsilon_{nL}$ This shift may be related to the corresponding hadron nucleus scattering parameter A_L, arising in the effective range expansion. The relation comes via an expansion in A_L/B^L, as the Bohr radius B is usually much larger than the lengths characteristic for the nuclear interactions. For the S waves

$$E_{nS} - \epsilon_{nS} = \frac{2\pi}{\mu}|\psi_n(0)|^2 A_0(1 - \lambda A_0/B) \tag{1}$$

is known as Trueman formula [11]. The electromagnetic energy ϵ_{nS} is composed of the Bohr energy with corrections for relativity and the deuteron electric polarisability. In the $1S$ state $\lambda = 3.154$,$B_{\bar{p}d} = 43.2 fm$, $A \approx 1 fm$ and the second order term in Eq.(1) makes a few percent correction. Such corrections are negligible in higher angular momentum states and a simpler relation, [12], is sufficient

$$\Delta E_{nL} = \epsilon^o_{nL}\frac{4}{n}\Pi^L_{i=1}(\frac{1}{i^2} - \frac{1}{n^2})A_L/B^{2L+1}. \tag{2}$$

These formulas generate the values shown in table 1.

2.1 A formula for the antiproton-deuteron scattering length

In this section the threshold $\bar{p}d$ scattering parameters A_L are calculated. Later this method is extended to the P and D parameters in He. The fine structure is calculable

Table 1: Level shifts in antiprotonic deuterium, [keV] for S and [eV] for P states. Third column gives the extracted $\bar{p}d$ scattering parameters and an absorptive length obtained from $\bar{p}d$ scattering [13].

level	$\delta E - i\Gamma/2$	$A_L[fm^{2L+1}]$
$1S$	1.05(25)-i0.55(37) [3]	0.71(16)- i0.40(27)
S		- i0.62(7) [13]
$2P$	243(26) -i245(15) [3]	3.15(33) - i3.17(19)

but at this moment a meaningful discussion may be done only with a spin and isospin averaged A_L. These are obtained by a summation of the $\bar{p}d$ multiple scattering series into a quasi-geometric series. For a full explanation of the method we refer to [14, 15], it compares successfully with exact calculations [18]. The multiple scattering series for scattering on two nucleons labeled n,p below the deuteron breakup is

$$\begin{aligned} T &= t_n + t_p + t_n G_0 t_p + t_p G_0 t_n + t_n G_0 t_p G_0 t_n + t_p G_0 t_n G_0 t_p \\ &+ (t_p + t_n) G_{NN} (t_p + t_n) + (t_p + t_n) G_{NN} (t_p + t_n) G_{NN} (t_p + t_n) + ... \end{aligned} \tag{3}$$

where t_i are $\bar{p}N$ scattering matrices, G_0 is the free three-body propagator and $G_{NN} = G_0 T_{NN} G_0$ is that part of the three-body propagator which contains the nucleon-nucleon scattering matrix T_{NN}. The deuteron scattering amplitude is determined by an average

$$T_{\bar{p}d}(E, L) = < \psi_d j_{\bar{p}}^L \mid T \mid \psi_d j_{\bar{p}}^L > \tag{4}$$

where ψ_d is the deuteron wave function and $j_{\bar{N}}^L$ is the Bessel function for an L-th partial wave in the antiproton-deuteron system. The $\bar{p}d$ scattering parameters are obtained in the limit $p \to 0$ of the expression $A_L = -T_{\bar{p}d}(E, L)/[p^{2L}(2\pi)^2 m_{\bar{p}d}]$ where p is the relative momentum and $m_{\bar{p}d}$ is the corresponding reduced mass. The normalization is related to a similar normalisation of the two body matrices. For S waves $t = -a(E)/[(2\pi)^2 \mu_{\bar{N}N}]$ where $a(E)$ is the scattering length. For P waves one needs a gradient operator which acts on the relative $\bar{p}N$ coordinates $t = -3b(E)/[(2\pi)^2 \mu_{\bar{N}N}]\nabla\nabla$ where $b(E)$ is the scattering volume. These matrices correspond to the four basic antiproton amplitudes of interest

$$f_{\bar{p}N} = a_N(E) + 3b_N(E)\mathbf{q}\mathbf{q}' \tag{5}$$

where N stands for the proton or neutron and $\mathbf{q}, \mathbf{q}'$ are the CM momenta before and after the collision.

The leading, single collision $< t_p + t_n >$ term yields the impulse $\bar{p}d$ scattering parameters

$$A_L = \frac{\mu_{\bar{p}d}}{\mu_{\bar{p}N}} \frac{1}{[(2L+1)!!]^2} [(\bar{a}_{\bar{p}p} + \bar{a}_{\bar{p}n}) < (r/2)^{2L} >_d + (\bar{b}^1_{\bar{p}p} + \bar{b}_{\bar{p}n})(\alpha + \beta < (r/2)^{2L-2} >_d] \tag{6}$$

where $< (r/2)^{2L} >_d$ is the $2L$-th radial moment of the deuteron expressed in terms of the relative coordinate r and lengthy numerical factors α, β come from the derivative P-wave term. In the limit of heavy nucleons, which is not used here, these numbers may be found in [15]. Energy averaged values of a and b arise in Eq.(6)

$$\bar{a}(E) = \int a\left(-E_d - \frac{p^2}{2m_{N,\bar{N}N}})\right) \mid \tilde{\phi}_d^L(p) \mid^2 d\vec{p}, \quad (7)$$

as a result of the spectator nucleon recoil. The extent of the involved energies is determined by the nucleon binding and the Bessel transforms of the deuteron wave function

$$\tilde{\phi}_d^L(p) = \int \psi_d(r) j_L(pr/2) r^2 dr \quad (8)$$

These energies cover some unphysical subthreshold region. The relevant distributions given by Eq.(7) peak around -12, -7 and -5 MeV for $L = 0, 1, 2$ correspondingly. For heavier nuclei and stronger nucleon bindings the energies of interest are shifted further away from the threshold. That gives the chance to study the energy dependence of $\bar{a}(E)$ and $\bar{b}(E)$. A partial sum of the series (3) is obtained by the first order geometric approximation

$$A_L^1 = \frac{A_L}{1 - \Omega - \Sigma} \quad (9)$$

with $\Omega =< t_p G_0 t_n + t_n G_0 t_p > / < t_p + t_n >$ and $\Sigma =< (t_p + t_n) G_{NN} (t_p + t_n) > / < t_p + t_n >$. This partial sum is 1% accurate in $2P$ states while in the $1S$ states a next step is needed to reach that precision. We also use these formulas to describe the $2P$ and $3D$ levels in antiprotonic He, expressing G_{NN} by a simple separable nucleon-core interaction .

3 The results

3.1 An extraction of the $\bar{p}N$ subthreshold amplitudes

The formalism of the previous section is now used to extract the averaged absorptive lengths Im $a(E)$ and absorptive volumes Im $b(E)$ from the d and He data. The solution is not fully determined. For each atom one has four parameters Im a, Im b, Re a, Re b and three atomic data - the lower shift and two level widths. In two cases, d and ^{4}He, these are supplemented by the S wave Im A_0 lengths [13]. The best fit for absorptive parameters is possible but the real parts cannot be determined. The results are summarised in fig.1 and compared with the updated Paris model calculation [16]. This semi-phenomenological model is based upon all available scattering data. More precisely, the quantities plotted are the half off-shell amplitudes $a, b(q = 0, E, q(E))$. Two findings are of interest.

First, there is an enhancement of the P wave absorptive amplitude just below the threshold. Within the model it corresponds to a quasi-bound $^{31}P_1$ state. Another type of threshold enhancement was recently found by the BES collaboration in the final $\bar{p}p\gamma$ states obtained in the J/ψ decays [19]. That enhancement may be attributed to the $^{11}S_0$ quasi-bound (or virtual) state which is also generated by the Paris model. However, the statistical weight of this state is too small to produce a clear signal in the atomic data. Second, there is an increase of the S wave absorption down below the threshold. Both

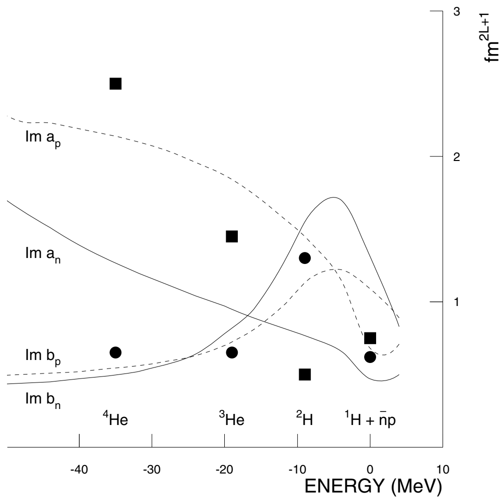

Figure 1: The absorptive parts of averaged subthreshold amplitudes calculated with Paris model: dotted lines - $\bar{p}p$, continuous lines - $\bar{p}n$. The lengths are denoted as a_p, a_n the volumes b_p, b_n. The $b_p/2 + b_n/2$ should be compared to the circles which give the average scattering volumes extracted from d, ^{3}He and ^{4}He. In the same way the extracted scattering lengths given by the squares are to be compared to $(a_p + a_n)/2$. The experimental results at the threshold come from the $\bar{p}p$ atom and $\bar{n}p$ scattering experiment [17].

these effects are fairly well understood in terms of the model, although the threshold result should be improved.

One can continue this analysis and cross the threshold from below to study the absorption measurements of $\bar{p}$ stopped in bubble chambers [9, 10]. The ratios of S wave absorption obtained in this way and given in table 2 are well reproduced by the Im a_n, Im a_p given in figure 1.

3.2 On the effects of $\Lambda(1405)$ and $\Sigma(1385)$ on Kd atomic levels

The significance of $\Lambda(1405)$ in the optical potential studies of K^- mesonic atoms is well known. It is the main factor that determines the mechanism of nuclear attraction. While in heavy nuclei the properties of $\Lambda(1405)$ may be changed by the medium they should be seen clearly in the $1S$ level of the Kd atom. However, the knowledge of subthreshold KN amplitudes is uncertain and detailed calculations depend on the way the KN data is parameterised. This is exemplified by calculations from ref. [18] given in table 3. Different types of amplitude parametrisations: constant lengths, K-matrix, separable potential fitted to the same data differ in the $1S$ level width by some 15%. The experiment could be helpful in that respect. The scattering lengths in table 3 seem outdated now [20, 21] but

Table 2: The experimental antiproton capture ratios $R_{n/p} = \sigma(\bar{p}n)/\sigma(\bar{p}p)$ extracted in states of flight at very low energies.

Element	R_{np}	method	state	reference
d	0.81(3)	chamber	stopped	[9]
^3He	0.47(4)	chamber	stopped	[10]
^4He	0.48(3)	chamber	stopped	[10]

the problem remains. It becomes more serious in He due to a rather distant extrapolation. The analysis performed in the last section for $\bar{p}$ may be repeated in the K case when both the Kd and KHe data are available. That seems possible even if the anomalous level shift in ^{4}He [22] is confirmed.

Table 3: The $1S$ Kd level shifts, comparison of several parametrisations of the same KN data. Taken from ref. [18]

	CSL	K-matrix	Separable
a_{Kp} [fm]	-0.623+i.763	-0.663+i0.665	-0.645 +i0.665
a_{Kn} [fm]	0.322+i.748	0.264+i.570	0.32+i0.70
$\epsilon, \Gamma[KeV]$	-0.50, 1.02	-0.49, 0.89	-0.50, 0.98

An interesting and open question is: what is the connection of the anomalous 2P shift in ^{4}He to the P wave resonance $\Sigma(1385)$. We believe it is strong, [8], but proper calculations are hard. At this moment we look for effects of $\Sigma(1385)$ in upper atomic levels. These, perhaps, may be detected by the cascade intensity balance. The calculations based on the S wave amplitudes from ref. [6] and SU(3) coupling to the P wave $\Sigma(1385)$ [8] are presented in table 4. These are not very optimistic. The chance to observe the $2P \to 1S$ transition in deuterium is low. The $\Sigma(1385)$ contributions to the $2P$, $3D$ widths are sizable but these widths would be difficult to extract from the cascade balance. The $3D$ level width in He, strongly affected by $\Sigma(1385)$, seems a bit easier to detect.

Table 4: The upper widths in Kd and K^4He levels

	Kd, $\Gamma(2P)$ [meV]	Kd, $\Gamma(3D)$ [μeV]	KHe, $\Gamma(3D)$ [meV]
Absorptive, S wave	26	$10 * 10^{-2}$	$2 * 10^{-2}$
Absorptive, S + P	32	$13 * 10^{-2}$	$3.8 * 10^{-2}$
Radiative	.32	32	.57

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