# Numerical modelling of the RR Lyrae instability strip

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## Abstract

Selected results are presented of an extensive model survey of the radial pulsation of HB stars using a turbulent convective (TC) code. The main goal of the investigation was to systematically map the possible pulsational modes in the classical instability strip. This contribution focuses on (i) the slope of the fundamental mode blue edge and (ii) double-mode RR Lyrae stars. The importance of the interplay between mode-selection effects and stellar evolution is emphasized in both cases.

## Introduction

Two main problems are addressed: (i) The discrepancy between the empirical and theoretical RR Lyrae fundamental (F) blue edges (Jurcsik 1997). Neither radiative nor convective models could explain the shallow slope of the blue edges of the F and O1 instability regions (Kolláth et al. 2000). It is well known from the early suggestion of van Albada & Baker (1973) that in the case of classical pulsators 'either-or-regions' exist on the HRD. The width of this hysteresis region can be several hundred K. As stars of different parameters pass through this region, mode selection can bend the *average slope* of the F blue edge significantly. The main motivation was to investigate this scenario by performing a large-scale nonlinear survey of radial RR Lyrae pulsation. (ii) Benefiting from the mode selection results we carried out a comprehensive investigation of nonlinear double-mode (DM) models. Since the first claim of successful modelling of double-mode pulsation (Feuchtinger 1998), no systematic exploration of the double-mode models has been published except some limited attempts (Szabó et al. 2000). These two issues are discussed within the common framework of the connection of pulsation and evolution.

## The method

In order to explore the mode selection characteristics the Florida-Budapest code (Kolláth et al. 2002) was used, which is a one-dimensional, nonlinear hydrocode including turbulent convection. Hydrogen content was set to be $X = 0.75$. Turbulent convection parameters were also taken from Kolláth et al. (2002). Model sequences with the following parameters were computed: $M = 0.50, 0.55, 0.60, 0.65, 0.71, 0.77, 0.82, 0.87 M_\odot$, $L = 40$, 50, 60, 70 $L_\odot$, and $Z = 10^{-4}$, $10^{-3}$, $4 \cdot 10^{-3}$; where only $T_{\text{eff}}$ was varied. To reduce significantly the computational cost, time-dependent amplitude equations were fitted to the sequences (Szabó et al. 2004). This allowed the interpolation of the limits of different pulsational states within a sequence. This way a large grid resulted, containing mode selection information throughout the relevant regions of the parameter space. We stress that the selection of turbulent parameters is not unique. The application of a different TC parameter set usually

Table 1: RR Lyrae fundamental blue edge slopes derived by different methods, assuming a linear relation.

| *method* | *slope* $(1\sigma)$ | *source* |
|---|---|---|
| **empirical** | **-5.40** | Jurcsik  1998 |
| convective, linear | -13.34 | Kolláth et al.  2000 |
| convective,  nonlin.,  Dorman  ev.  tracks | -3.91 (0.51) | this  paper |
| convective,  nonlin.,  Demarque  ev.  tracks | -3.96 (0.25) | this  paper |
| convective,  nonlin.,  Padova  ev.  tracks | -4.25 (0.47) | this  paper |

causes a temperature shift in instability strip structures, but the fundamental blue edge slope is not affected by this shift.

## Mode selection

In the following we simply denote the union of DM and F/DM region by DMR, if not stated otherwise. (1) The following combinations of stable fixed points were present in our models: single stable fixed point: F, O1, or DM; two simultaneously stable fixed points, i.e. hysteresis regions: F/O1 and F/DM. (2) The topology of the instability strip is similar but not identical for different metallicity and luminosity values. (3) F/O1 can be seen across the whole investigated range of $Z$ and $L$. (4) DMR is also present at all $Z$ and $L$ values, generally with a small width: $10 - 60K$. At low $L$ one can find a mostly pure DM area, while at higher luminosity F/DM appears immediately at the lower temperature side of DM. (5) DMR and F/O1 have a strong connection, both are situated between pure F and O1 regions. For fixed $L$ and $Z$ one can see F/O1 at low mass and DM at higher mass. The transition mass increases with increasing luminosity.

## Fundamental blue edge

Combining the mode selection and evolutionary information one can construct a theoretical $T_{\mathrm{eff}} - L$ diagram containing RR Lyrae stars pulsating in the fundamental or first overtone mode. Then the blue edge slope can be easily determined. To this end a synthetic horizontal branch stellar population was generated with uniform mass, metallicity and age distributions. Three HB evolutionary track sets were applied: (1) Dorman-tracks (Dorman 1992), (2) Demarque-tracks (Demarque et al. 2000) and (3) Padova-tracks (Girardi et al. 2000). For the parameter sets of each individual model star an evolutionary track and its pulsational properties were determined by interpolation of the evolutionary and pulsational grids, respectively. RRd stars could be omitted thanks to the narrow double-mode regions. The short time delay during switching from one pulsational state to another (Buchler & Kolláth 2002) could be safely ignored as well. 10 000 Monte-Carlo iterations were performed for all the three evolutionary track sets. The slopes of the linear fits are listed in Table 1. The distribution around the mean slope is well approximated by a Gaussian, standard deviations $(1\sigma)$ of these distributions are listed in parenthesis.

The presented method provides much better agreement between empirical and theoretical slopes than previous simulations did. No significant variations were found using different evolutionary computations. From the observational point of view the fundamental blue edge is equivalent to the envelope of the RRab stars. We emphasize that from the theoretical side F blue edge always means the blue edge of F/O1, and the O1 red edge is defined by the red edge of F/O1. Evidently, the fundamental blue edge established by this new method is also defined as the blue envelope of the fundamental pulsators, because of the combined

effect of mode selection and evolution. It is worth mentioning that the *L,M,Z*-dependence of the F/O1 edges are all important factors, and the relevance of both mode selection and evolution in blue edge modelling were confirmed by simple tests and our earlier work (Szabó et al. 2002).

## Double-mode pulsation and evolution

Synthesis of existing evolutionary calculations and mode selection maps led to surprising results. The most important feature is the narrow mass range, ($M = 0.745 \pm 0.010 M_\odot$, $Z = 0.0001$) where the DMR is crossed by evolutionary tracks. For $Z = 0.001$ the mass range is similar, but at lower mass regime ($M = 0.665 \pm 0.010 M_\odot$). At $Z = 0.004$ no tracks cross the DMR. This trend is in good agreement with the results of Popielski et al. (2000), although they get somewhat higher mass and larger mass range. We emphasize again that altering the TC parameters affects instability strip structures. In this case the result may be an enlarged DM mass range, and/or increased DM mass. Petersen-diagram supports the narrow mass-range (Szabó et al. 2004). If evolution is taken into account, i.e. tracks crossing the DMR, then the possible DM region on the Petersen-diagram is approximately confined to the distribution of RRd stars. This is the first attempt to reproduce the RR Lyrae Petersen-diagram on the basis of nonlinear double-mode models. It is important to note that if only the F/DM region exists, then only redward evolution produces DM pulsation. Although we encountered this scenario we are not in the position to exclude the possibility that blueward or both blue- and redward evolution produce DM stars. A small shift in DM and F/DM positions or in evolutionary tracks may change the situation. This clearly demonstrates the delicate interplay between mode selection and evolutionary effects in determining the possible parameter range of double-mode RR Lyrae pulsation.

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