

THE AURORAL KILOMETRIC RADIATION FROM URANUS AND ITS MAGNETOSPHERIC IMPLICATIONS

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Abstract

The Gulkis and Carr (1987) emission beam model for the kilometric radio emission from Uranus and the types of information regarding the Uranian inner magnetosphere that may be derivable from its application are discussed. A modeled electron density profile is presented. The manner in which the hollow-cone beam from each element of the distributed auroral-zone source varies with frequency is determined. It is suggested that the observed effects might be due to refraction by a smooth component of the electron density variation with distance, together with multiple small refractions by electron density inhomogeneities along the ray path.

1. Introduction

Radio emission from Uranus was observed by Voyager 2 over the approximate frequency range 20 to 850 kHz (Warwick et al., 1986). Most of the activity occurred after closest approach, when the spacecraft was above the dark side of the planet, although emission at the lower frequencies was observed before the spacecraft left the daylit side. At most frequencies at which activity was observed, the intensity varied in a periodic fashion indicative of beamed emission from quasicontinuous sources, the beams rotating with the planet. This modulation pattern was in fact used to determine the planetary rotation period (Warwick et al., 1986; Desch et al., 1986). The radiation was predominantly elliptically polarized in the left hand (LH) sense, although there were a number of instances of right hand (RH) polarization. Gulkis and Carr (1987), assuming that the LH-polarized emission arose from the same process that is responsible for the terrestrial Auroral Kilometric Radiation (AKR) and was emitted only from above the darkside auroral zone, were able to construct a geometrical beaming model that could be adjusted to reproduce much of the structure of the observed intensity patterns over a wide range of frequencies. The Gulkis and Carr paper will be referred to as Paper 1. In the present paper, after a brief discussion of the model, we indicate some ways in which it might be used to deduce new information on the Uranian magnetosphere.

2. The emission model

We present here a brief description of the emission model and the results obtained with it; a full treatment is given in Paper 1. It is assumed that this main component of the Uranian kilometric radiation, which we shall refer to as UKR, occurs principally in the extraordinary mode (X mode) at frequencies slightly above the local electron cyclotron frequency, and is beamed within hollow-cone patterns that are fixed relative to the magnetic field. We assume the emission to be produced by auroral electrons guided by field lines connecting the magnetotail with the auroral zone surrounding the south-magnetic (negative) pole on the dark side of the planet. X-mode emission escaping from the vicinity of a south magnetic pole is elliptically polarized in the LH sense. The emission mechanism is assumed to be the relativistic cyclotron resonance (cyclotron maser) mechanism (Wu and Lee, 1979; Omidi et al., 1984) which is generally accepted to be the source of AKR.

One of the important similarities of the observed Uranian emission to AKR was with regard to the morphology of the spectral distribution of peak flux density. The peak flux density spectra of the radiations from Uranus, Earth, Saturn, and Jupiter, normalized to a distance of 4 A.U., are shown in Figure 1. The curve for Jupiter has not previously been published; it represents data recently obtained from the Florida Edited Voyager–Jupiter Data Set. It is apparent that the curves for Uranus, Earth, and Saturn are similar. All three display a low frequency turnon near 20 kHz, peak emission near 200 kHz, and a high frequency cutoff, suggesting a common emission mechanism. The curves are consistent with the idea that the radiation from both Uranus and Saturn arises from an emission mechanism of the AKR type. Although Jupiter also emits strongly in this frequency band, its curve displays no well-defined peak nor high frequency cutoff at these frequencies. It appears likely that although a similar type of emission mechanism may be involved, the morphology of the Jovian spectrum is entirely different because of the much stronger magnetic field and greatly different plasma distribution.

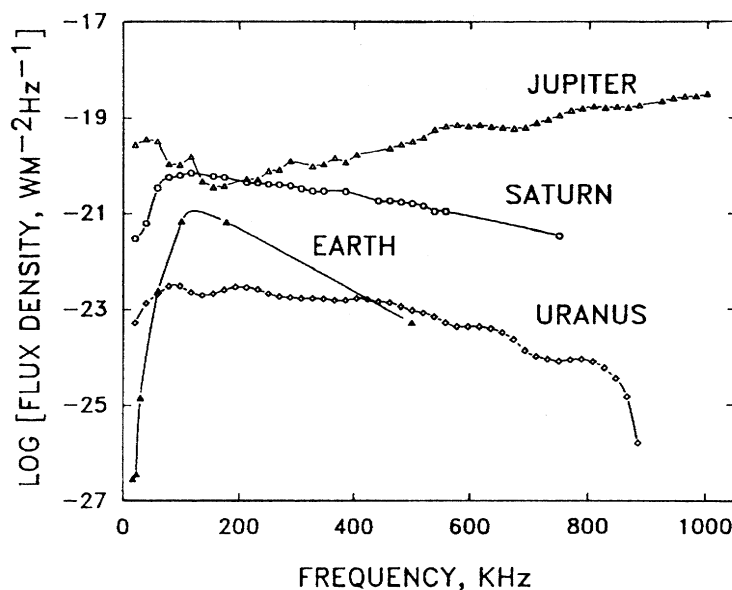


Fig. 1: Low frequency radio power spectra of Uranus (Gulkis and Carr, 1987), Earth (Gurnett, 1974), Saturn (Carr et al., 1981), and Jupiter (previously unpublished data from Voyager 1). The curves are for peak values of the flux densities, which have been adjusted to the distance 4 AU.

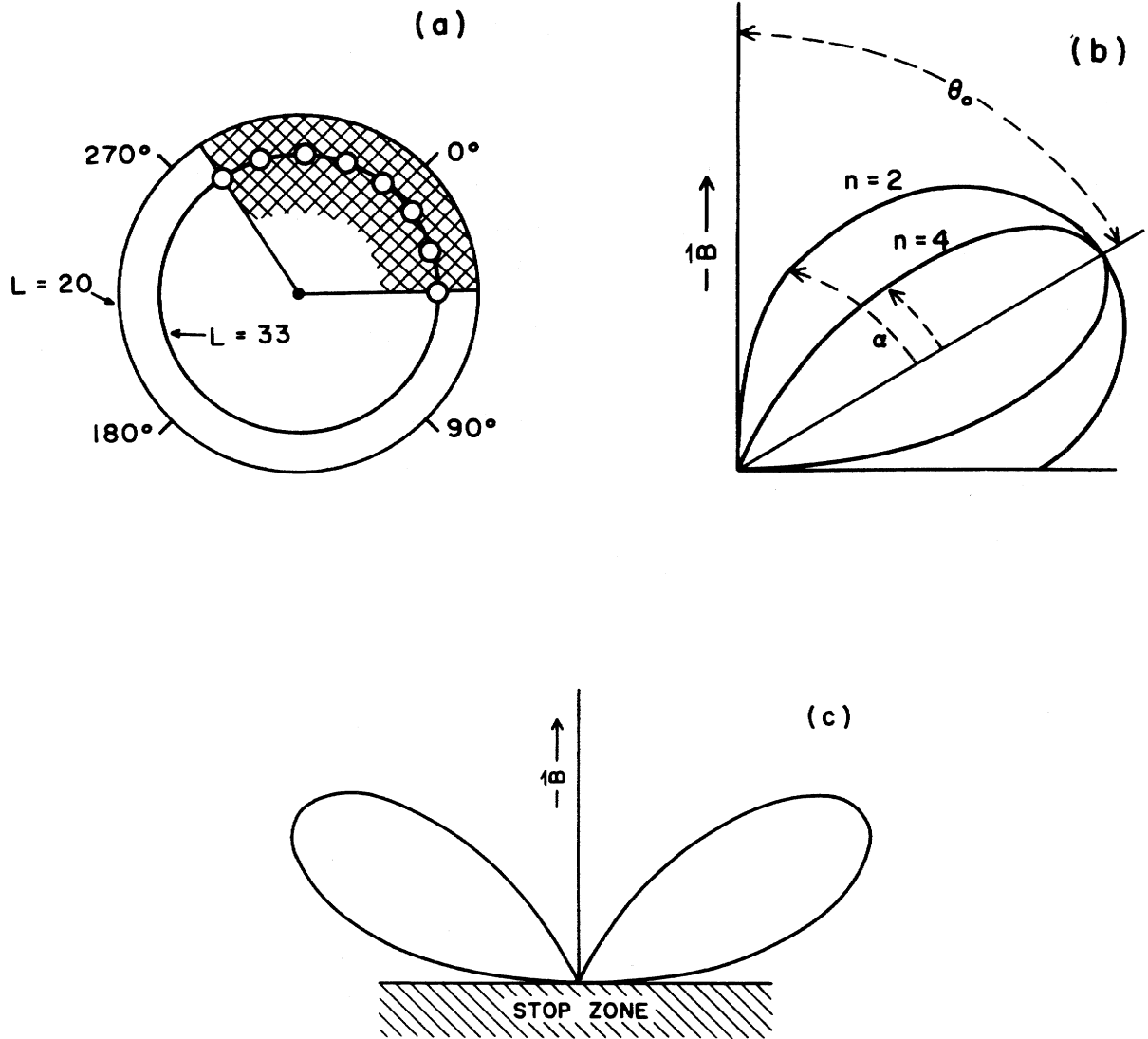


Fig. 2: The beam model. a) South-magnetic polar projection of the radio ring (indicated by $L = 33$) for a particular emission frequency. The active sector is the portion of the ring within the cross-hatched region. The eight small circles are the subsources that have been used to approximate the emitting ring sector, which in turn is an approximation to the more widely distributed true emission region suggested by the cross-hatching. Magnetic longitude is indicated outside the $L = 20$ shell, which approximately bounds the UV auroral region (at a much lower altitude). b) Hollow-cone beam generating functions given by Equation (1) for an opening half-angle θ_0 of 60° and two values of the thickness parameter, n . (c) Section through the hollow-cone beam from a single subsource, in a plane containing the cone axis.

The south-magnetic pole of Uranus was in uninterrupted darkness at the prevailing phase of the planet in its solar orbit, since the rotational angular velocity vector was only 10° from the direction of the sun and the south-magnetic pole was about 45° of latitude from the rotational pole on the side opposite the sun. The Offset Tilted Dipole (OTD) model of the magnetic field (Ness et al., 1986) was assumed. In this field model, the magnetic dipole is located nearly on the rotational axis at a distance of $0.31 R_U$ from the center of the planet in the negative direction of the angular velocity vector; the dipole moment vector is tilted 60° from the rotational angular velocity vector.

An ultraviolet aurora was observed during the encounter (Broadfoot et al., 1986) within a region later shown to surround the south-magnetic pole and to be bounded approximately by the $L = 20$ magnetic shell of the OTD field model. For simplicity, the radio-emitting auroral electrons were assumed to be confined to a single magnetic shell, i.e., the $L = 33$ shell. The foot of this shell is about 2° of latitude inside the auroral zone boundary, which was about 18° in diameter. The radio emission at a given frequency was thus assumed to arise from a region close to the circle defined by the intersection of the $L = 33$ magnetic shell with the magnetic field intensity contour surface on which the electron cyclotron frequency equals the particular emission frequency. We shall refer to this circle as the radio ring.

We initially assumed that the entire radio ring radiates as a distributed incoherent source. It was later found necessary to restrict the radiating region to a particular sector of the ring, which we shall refer to as the active sector. To facilitate computation, we replaced the active sector by eight equally spaced subsources of equal strength radiating incoherently with respect to each other. The emission from each subsource was assumed to be beamed within a thick sheet that is curved about the field line through the subsource. The directions of maximum emission in this sheet define a cone, opening in the negative B direction, the axis of which is tangent to the field line. Thus the composite beam of the entire active ring sector is approximated by the sum of the overlapping hollow-cone beams from the eight subsources. Figure 2a is a plane section containing the radio ring (indicated by $L = 33$). The eight subsources representing the active sector are indicated by small circles. The actual radiating region is suggested by the crosshatched area in the figure. The active sector angular width and location (in terms of the magnetic longitude scale in the figure) are adjustable parameters of the model. The opening angle and angular thickness of the hollow-cone beam from the individual subsource are also adjustable parameters. The subsource beam pattern is specified by means of a generating function of the type illustrated in Figure 2b, and is defined in the first quadrant by the relation

$$f(\theta) = \frac{1 + \cos[n(\theta - \theta_0)]}{2} \quad (1)$$

where $f(\theta)$ is the relative power per unit solid angle radiated in any direction making an angle θ with respect to the negative direction of the B vector at the subsource, θ_0 is the cone opening half-angle, and n is a parameter that determines the beam thickness. A further restriction on $f(\theta)$ is that it is zero outside the first quadrant. The beam thickness α (in Figure 2b), from the lobe axis to the inner half-power direction, depends on the

value of n ; for the illustrated beams having n values of 2 and 4, the values of α are 45° and 22° , respectively.

The beam from the individual subsource is defined by rotating the generating function $f(\theta)$ about the negative- $\mathbf{B}$ vector at the subsource. Figure 2c is a section (in any of the planes containing this negative- $\mathbf{B}$ vector) through the hollow-cone beam generated in this way by the $n = 4$ curve of Figure 2b. X-mode radiation at a given frequency cannot propagate in a region where the cyclotron frequency equals or exceeds that frequency; such a region is a stop zone. Each subsource is assumed to lie just outside the stop zone boundary. This is illustrated in Figure 2c. In Figure 2b, it is the presence of the stop zone boundary that causes the abrupt termination of the $n = 2$ curve where it meets the horizontal axis. Thus far we have assumed that at each frequency the stop zone boundary is perpendicular to the magnetic field everywhere in the corresponding radio ring. This, however, is not precisely correct. Contour surfaces of constant magnetic field intensity (of which the stop zone boundary is an example) are not necessarily normal to the field lines. The departure from perpendicularity is about 5° in the 78 kHz radio ring. We are currently investigating the possibility of observable polarization effects due to this non-perpendicularity.

Each part of Figure 3 is a section through the modeled emission patterns of a pair of sub-sources assumed to be diametrically opposite in the radio ring, for each of two frequencies. The upper and lower parts of the figure are for two pre-encounter times that were a half rotation period apart, the times having been selected so that the spacecraft, the rotation axis, and the magnetic dipole axis were very nearly in the plane of the drawings. The sunlit hemisphere is the upper half of the planet in each part of the figure. It should be noted that in the upper part of the figure, 78 kHz radiation is reaching Voyager while 212 kHz radiation is not, and that a half rotation period earlier or later (lower part of the figure) no radiation at either frequency reaches the spacecraft. This accounts for the fact that 78 kHz radiation, intensity-modulated by the planetary rotation, was observed before encounter while almost no pre-encounter radiation was received at 212 kHz.

As has been stated, beam and active sector parameters which were adjusted arbitrarily to produce the best fits to the observed plots of emission intensity as a function of time at each frequency included (1) the subsource hollow-cone beam opening half-angle α and thickness factor n , and (2) the two values of magnetic longitude defining the boundaries of the active sector of the radio ring (or alternatively, the midsector longitude and the angular width of the sector). Additional adjustable parameters were the relative strength of the constant background intensity level that was added to the modeled modulated intensity curve, and the scale factor for the resulting plot. These parameters were varied until a reasonably good fit of the modeled to the observed intensity curves was obtained at each of three representative frequencies, 78, 231, and 481 kHz.

In Figure 4a, the observed 78 kHz data (represented by points) are compared with one of the best-fitting plots (smooth curve) obtained from the model. Each data point is

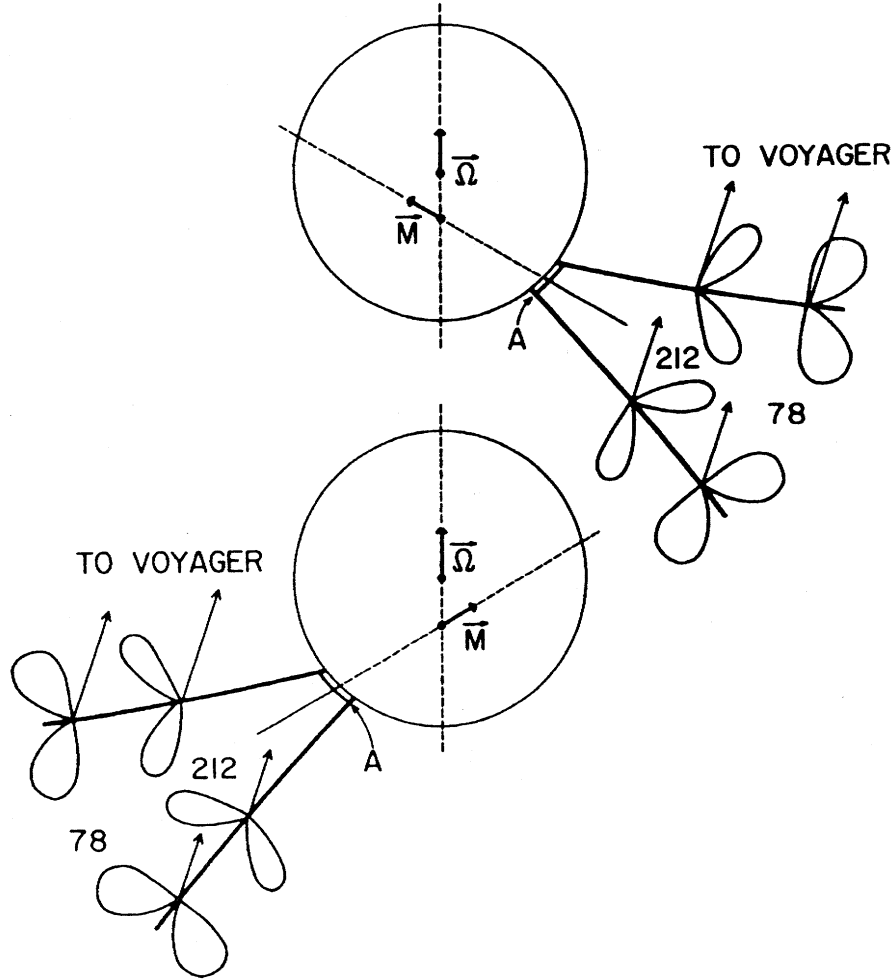


Fig. 3: Sectional diagrams in the plane containing the angular velocity and magnetic moment vectors, $\vec{\Omega}$ and $\vec{M}$, respectively (neglecting their relatively small offset). Field lines for the $L = 33$ shell are shown. For two particular times before closest approach and two frequencies, 78 and 212 kHz, the bilobed patterns represent the emission from two subsources assumed to be diametrically opposite in the radio ring. The times differ by approximately half a rotation period. The auroral zone is indicated by A.

Frequency kHz	Beam maximum angle θ_0	Beam thickness		Active sector magnetic longitudes		
		n	α	Extent	Midpoint	Width
78	60°	1	90°	-160° to +150°	-5°	310 °
231	35°	3	60°	-70° to +50°	-10°	120 °
481	50°	6	30°	-70° to +50°	-10°	120 °

Table 1: Model subsource beam parameters

a 20-min average. The values of the adjustable parameters of the model for this and subsequent plots are given in Table 1. It is apparent that the modeled curve well reproduces the pre-encounter radiation that is intensity modulated by the rotation of the planet. The phase of the pre-encounter modulation by the model is correct, but the height of the last peak before closest approach (closest approach was at DOY = 24.8) is only about half

that of the actual data. As already indicated in connection with Figure 3, since the spacecraft approached from the sunward side of the planet the pre-encounter radio emission was spill-over from the darkside radio ring. The timing of the abrupt rise of the modeled curve at the start of the main peak (embracing the time of closest approach) is correct, as is the general shape of the subsequent slow decay in intensity over the day of two following closest approach. While the post-encounter modulation peaks are weaker and of more complex structure than those before encounter, the modeled curve nevertheless tends to follow them approximately.

In Figure 4b, the 231 kHz data points and the corresponding modeled curve for the same time period are compared. It is seen that the model allows no pre-encounter radiation, while the actual data indicate almost none. As demonstrated in Figure 3, the subsurface hollow-cone beam is now sufficiently thin that there is no pre-encounter spill-over from the darkside radio ring. The fit of the main peak in Figure 4b is reasonably good, as is that of the first two modulation peaks following it. Although the modeled curve continues to indicate the correct modulation phase beyond DOY = 27.0, it tends to underestimate the modulation amplitude. Figure 5a is a portion of Figure 4b for which the time scale has been expanded.

In Figure 5b, the 481 kHz data points and the corresponding modeled curve are compared for the same time period as that of Figure 5a. Again, neither the observed data nor the model indicate appreciable pre-encounter radiation. The time of onset of the main peak is correctly given by the model. The duration of the peak is approximately correct, but the model does not reproduce its structure. The post-encounter modulation peaks of the modeled curve in most cases correspond to apparent modulation peaks in the data, although the fit is not close.

A closer comparison of the 231 and 481 kHz post-encounter modulation peaks in the curves of Figure 5 reveals important information. It should be noted that there is a strong tendency for each of these post-encounter peaks in the 231 kHz observed data plot to be split into two shorter ones at 481 kHz. This is best illustrated by the first modulation peak following the main peak, at 231 kHz; its maximum occurs at about DOY 25.7. At 481 kHz at the same time there is a minimum between two peaks, each of which is of about half the duration of the 231 kHz peak. It appears at 481 kHz that a “bite” has been taken out of the 231 kHz peak. Our modeled curves reproduce the effect correctly, although the fit is not particularly close. We believe that this peak-splitting at the higher frequency is simply explainable in terms of the hollow-cone beam geometry. At DOY 25.7, the spacecraft had approached closest to the direction of maximum intensity of the 231 kHz hollow beam. But because the opening angle of the 481 kHz hollow-cone beam is larger, the spacecraft had already crossed the direction of maximum intensity of this cone and was inside where the intensity is low. In passing out of the hollow of the 481 kHz beam, the spacecraft again had to cross a direction of maximum intensity. Thus, two peaks were generated at 481 kHz during the same time in which one was generated at 231 kHz.

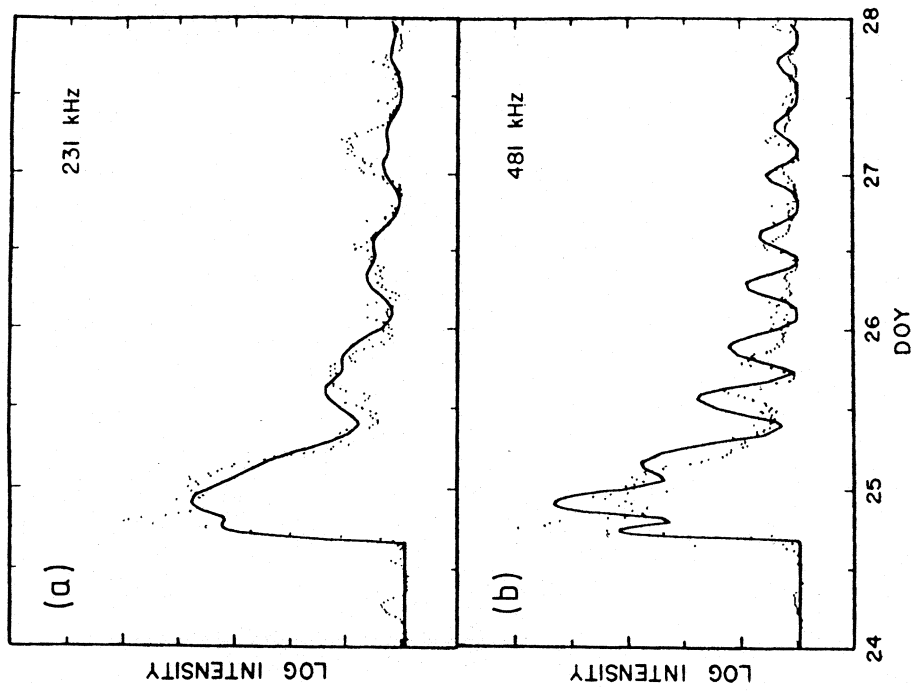


Fig. 5: Comparison of observed (dots) and modeled (solid lines) curves of intensity as a function of time, at the frequencies 231 and 481 kHz. DOY is day of the year, 1986.

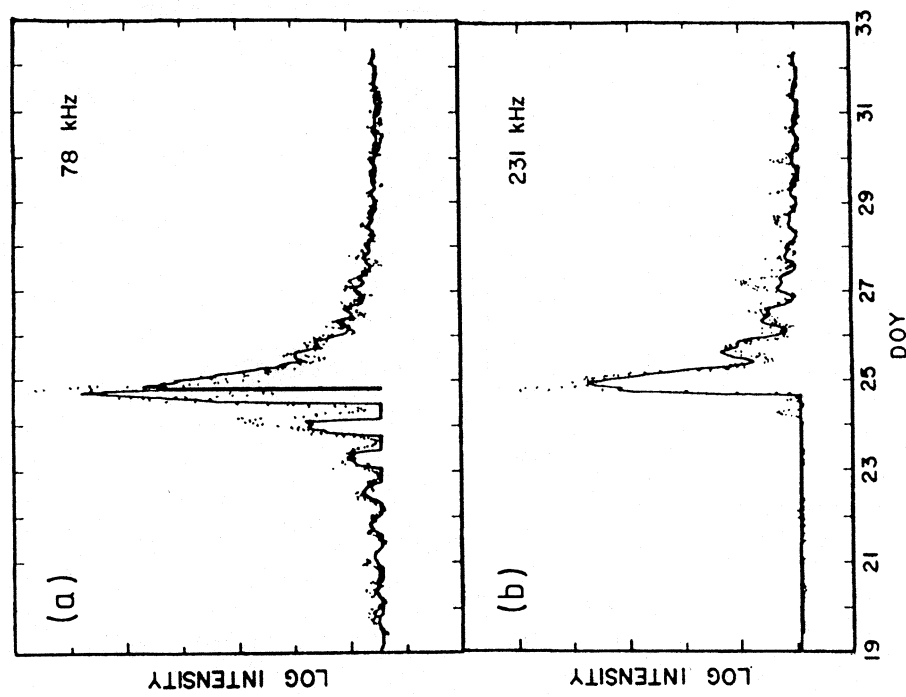


Fig. 4: Comparison of observed (dots) and modeled (solid lines) curves of intensity as a function of time, at the frequencies 78 and 231 kHz. DOY is day of the year, 1986.

3. A model plasma density distribution in the radio emitting region

A characteristic feature of the UKR radiation is the existence of well defined upper and lower cutoff frequencies (Figure 1). We make use of this feature to estimate the plasma density distribution in the emitting region. Our model assumes that the emission mechanism is similar to the cyclotron maser mechanism proposed for terrestrial AKR. The growth rate of electromagnetic waves in a cyclotron maser at resonance has been discussed on theoretical grounds by a number of authors (e.g., Gurnett and Anderson, 1981). It has been found that the ratio of the plasma frequency to the cyclotron frequency controls the growth rate of the amplified waves. Typically, the growth rate of an X-mode wave (of harmonic number 1) is a maximum when $f_p < 0.3f_c$. For higher values of f_p , the growth rate rapidly decreases. Experimentally, Calvert (1981b) found f_p/f_c to be less than 0.1 over the large frequency range within which AKR had been observed. This condition can exist over a wide range of altitudes in the region where the ionosphere merges with the background plasma of the magnetosphere. The crossover points at which f_p dips below 0.1 times f_c are thus believed to determine approximately the upper and lower cutoff frequencies for AKR.

It is reasonable to assume that a similar situation exists in the lower magnetosphere of Uranus. Cyclotron maser emission can presumably occur in those regions where the plasma-to-cyclotron frequency ratios along auroral field lines are favorable. We therefore assume that $f_p = 0.1f_c$ at the two points along an $L = 33$ field line at which f_c equals the observed upper and lower UKR cutoff frequencies, respectively. In Figure 6, the curve marked $\log(0.1 f_c)$ vs R indicates the cyclotron frequency variation, on the $L = 33$ field line, with respect to the distance from the dipole center. Since the dipole is offset considerably from the center of the planet, we also show two examples of corresponding curves (dashed curves in the figure) in which R represents distance from the center of the planet instead of from the dipole. The upper dashed curve is for the section of the $L = 33$ field line that passes closest to the center of the planet, and the lower dashed curve is for the section of the same line that is farthest from the center. Such curves for most of the other $L = 33$ field lines would fall between these two limiting curves.

We now assume that the plasma distribution along the field line is that due to a hot ionospheric component and an even hotter magnetospheric component, both distributed exponentially but with different scale heights. The peak electron number density of the ionospheric component is taken to be 10^5 electrons/cm³, close to that derived by Atreya (1984) for the Uranian auroral region. In arriving at this figure, Atreya assumed that 10^{11} watts of power is deposited in the solar-illuminated part of the ionosphere by 1 keV electrons. We further assume that the molecular weight of the upper atmospheric gas is that of hydrogen, and that the temperature is 1000 K. This gives a scale height of approximately 800 km for the hot ionospheric component. The plasma frequency corresponding to this ionospheric component along is consistent with the observed high-frequency cutoff of UKR near 800 kHz. However, the electron distribution falls off too sharply to explain the low-frequency UKR cutoff near 20 kHz. From this we infer that a higher-temperature plasma component must also be present. The source of these hotter electrons might be

secondaries produced by intense ion precipitation, although this is by no means certain. We find by trial and error that a suitable electron distribution would have a density of 1 electron/cm³ near the ionospheric peak and a scale height of approximately 1 R_U . The equivalent temperature is approximately 30 000 K, which corresponds to electron energies of a few eV. Electrons having this energy could not escape from the gravitational potential well, and would remain in the vicinity. Such a two-component plasma distribution accounting for both the observed low-frequency and high-frequency cutoffs of the UKR was therefore adopted. The plasma frequency and electron number density variation with distance from the center of the planet, calculated from this plasma model, are shown in Figure 6. Radio emission occurs for R values approximately between the two intersections of the f_p and $0.1 f_c$ curves in the figure. The two dashed curves can be used to estimate the uncertainty in the plasma density profile resulting from the fact that, at a given frequency, emission regions located on different field lines in the $L = 33$ shell are at different distances from the center of the planet. For each value of R (from the center of the planet) for which the values of f_p and N (electron number density) are given by the corresponding curve in the figure, the uncertainty in R is indicated by the length of the horizontal line segment between the intersections with the two dashed curves.

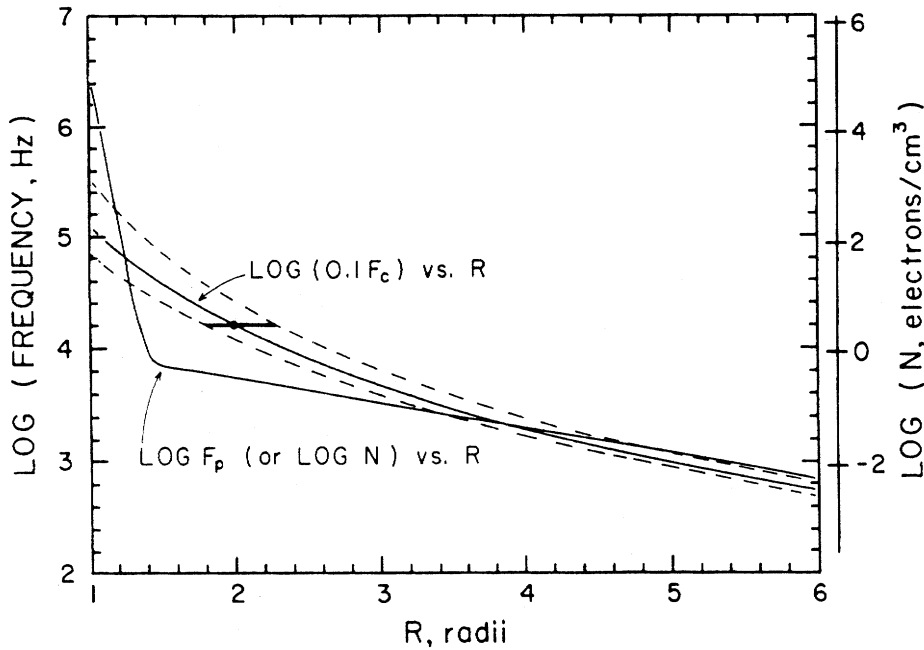


Fig. 6: Model curves of the logarithms of plasma frequency (f_p), electron number density (N), and 0.1 times the cyclotron frequency (f_c) in the $L = 33$ shell as functions of radial distance (R). In the f_p curve, R is the distance from the center of the planet. Although R in the $0.1 f_c$ curve is actually the distance from the dipole center, the approximation has been made that it too is the distance from the center of the planet. The resulting range of uncertainty in R is given by the horizontal distance between the two dashed curves. Thus the uncertainty in the value $R = 2$ corresponding to $\log f_p = 4.18$ is the length of the horizontal line segment through the indicated point at $R = 2$ on the $0.1 f_c$ curve. Radio emission occurs for R values approximately between the two intersections of the f_p and $0.1 f_c$ curves.

4. Other magnetospheric implications

The determination of the manner in which the subsource hollow-cone opening angle and thickness vary with frequency provides new information related to the emission mechanism and plasma environment of the source. Definite trends are apparent from our modeling at 78, 231, and 481 kHz. It is clear that there is a pronounced increase in the thickness of the conical-sheet beam as the frequency is decreased; the angle from the beam maximum to the inner half-power direction (α in Figure 2b) is 30° , 60° , and nominally 90° at 481, 231, and 78 kHz, respectively (the value at 78 kHz would be 90° if the beam generating function were not limited to the first quadrant, the beam actually remains above the half-power level everywhere inside the hollow). It is equally clear that at least between the two higher frequencies the cone opening angle decreases with a decrease in frequency; the angle from the beam maximum to the cone axis (θ_0 in Figure 2b) is 50° and 35° at 481 and 231 kHz, respectively. Although at 78 kHz the cone opening angle is nominally 60° , this angle is essentially undefined because the hollow of the hollow-cone beam has become nearly filled. We can summarize these two effects as follows: *Effect 1* – The thickness of the hollow-cone beam increases as the frequency is decreased, nearly filling the hollow when frequency is decreased to 78 kHz. *Effect 2* – At the higher frequencies for which the beam is thin enough to approximate a hollow cone, the cone opening angle decreases as the frequency is decreased.

The frequency dependence of both the above effects appears to be qualitatively consistent with the idea that they result from magnetospheric refraction. Refraction by the ambient plasma, which is greater the lower the frequency, would tend to make a hollow-cone opening angle smaller than it would have been without refraction. A possible explanation we suggest for Effect 2 is therefore that the angle θ_0 (Figure 2b) is somewhat greater than 50° at the source for all (or a wide range of) observed frequencies, but that refraction reduces it to 50° at 481 kHz and to 35° at 231 kHz. We further suggest the possibility that Effect 1 is caused by multiple refraction (scattering) by magnetospheric plasma inhomogeneities that are many wavelengths in size but represent only very slight enhancements of plasma density. At the higher frequencies such scattering thickens the hollow-cone beam only slightly, we assume, but the amount of ray bending at each encounter with one of these clouds becomes much greater as the frequency is reduced. At 78 kHz and below, multiple refraction has nearly eliminated the hollow-cone structure of the original beam, tending to make the beam isotropic. This tendency toward beam isotropy, however, is only for directions away from the stop zone; rays bent toward the stop zone will subsequently be refracted away from it before the boundary is reached.

A test of the consistency of our explanation of Effect 2 with our model plasma density curve in Figure 6 can be made by ray-tracing. It should be possible to determine whether the plasma density gradient derived from Figure 6 is sufficient to reduce θ_0 by refraction from somewhat over 50° to, for example, 35° at 231 kHz. This has not yet been attempted. A positive result would lend credence both to the role of refraction in reducing the cone angle and to the validity of the plasma density curve. On the other hand, if it is found that there is not enough refraction to account for Effect 2, it might be concluded that

1) the frequency variation of θ_0 is a characteristic of the source mechanism instead of resulting from refraction, 2) the plasma density curve of Figure 6 is incorrect, or 3) our determinations of θ_0 are incorrect. One possible source of error in θ_0 determination is in our use of the oversimplified OTD magnetic field model; steps being taken to reduce this error are discussed below. Insight into the plausibility of our hypothesis accounting for Effect 1 might be obtained by assuming a model magnetosphere containing clouds of slightly enhanced plasma density, and determining whether multiple refractions by these clouds could deviate rays sufficiently to produce the degree of isotropy observed at 78 kHz. If this result were positive, it would support our proposed method for obtaining information on the inhomogeneity of a region of the magnetosphere of Uranus.

Another potential source of magnetospheric information is the optimization (in comparisons of modeled with observed intensity plots) of the L shell value and the magnetic longitudinal location and width of the active sector of its radio ring, at each frequency. Such determinations would aid in the identification of the field lines along which the radio auroral electrons have arrived, and together with an improved field model, might permit the tracing of these field lines back toward the magnetotail region of solar wind electron capture. Regarding our choice of L-shell, we are confident only that the $L = 33$ shell is located in the right general vicinity. Little effort has been expended on optimizing its value, and less in ascertaining its possible frequency dependence. Our rough determinations of active sector widths and locations at three frequencies (Table 1), however, represent a start on this aspect of the radio-auroral field localization. We found no significant difference in the magnetic longitudes of the midpoints of the active sector at the three frequencies. In all three cases it was near 0° (actually -5° or -10°). This is the magnetic longitude that is closest to the darkside rotational pole, and nearly the closest to the antisolar direction. It appears that the field lines in this magnetic longitude region are the most centrally located of the $L = 33$ lines in the magnetotail, and perhaps for this reason are most active in producing auroral particles. The angular width of the active sector was much greater at 78 kHz than it was at 231 and 481 kHz, implying that electrons arriving along field lines closer to the midsector longitude reach much lower altitudes before radiating than do those at longitudes farther from the sector center.

All these avenues of investigation could perhaps provide valuable magnetospheric information if the radio modeling procedures were improved. We expect that the greatest improvement will be in the use of the Q3 magnetic field model instead of the OTD model. The Q3 field expansion contains quadrupole and octupole components as well as the dipole component (Connerney et al., 1987). It much more nearly approximates the true field than does the OTD model, particularly closer to the planet where the higher radio frequencies are emitted. Modification of our present beam modeling procedures are required when the Q3 field model is to be employed; for example, the loss of dipolar symmetry means that magnetic shells cannot be specified by their L values, and the radio ring is no longer circular nor flat. Work on these modifications is currently under way.

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