

EXTRAORDINARY MODE WAVES ON THE RESONANCE CONE DETECTED ON URANUS

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Abstract

Previously unidentified waveforms of decreasing frequency with time picked up in the Voyager 2 flyby of Uranus are a possible band of extraordinary waves on the resonance cone. The resonance cone effect causes only certain frequencies to be observed at certain angles from the source, and the frequency decreases as the angle decreases. The homogeneous plasma theory predicts an almost linear drop in frequency with time, and might be useful for estimating the distance and angle of the source. Inhomogeneities in the plasma cause variations on the homogeneous case, and create fine structure in the waveforms. They give arches to the waveforms, and certain forms of inhomogeneities can explain sudden drops in the frequency and changes in the slope of the waveforms at certain times, as is observed in the data. Furthermore, the multiple peaks in the frequency observed for given time in the waveform are well explained as the multiple peaks from the resonance cone interference structure that are predicted in the warm plasma theory for resonance cones.

1. Introduction

The recent Voyager-2 flyby of Uranus yielded a variety of plasma wave observation, such as electron plasma oscillations, whistler mode emissions, hiss and chorus (Gurnett et al., 1986). There were, however, a few emissions that originally could not be identified. One example of these types of emissions is shown in Figure 1. This mode has the characteristic that it rapidly drops down in frequency from its initial value in a matter of seconds. The second waveform shows a drop from $f_{max} = 1.1kHz$ to around $f_{min} = 0.6Hz$ in about 2 seconds. The local plasma and cyclotron frequencies observed at the time the wave was detected were:

$$f_{pe} = 11kHz \quad f_{ce} = 6kHz \quad (1)$$

An interesting explanation for this wave can be made through extraordinary mode resonance cones radiated from localized sources. These have been a major area of research in recent years (e.g. Fisher and Gould, 1971; Ohnuma, 1978 and references cited therein). What occurs is that under certain plasma conditions for a given wave frequency the electrostatic part of the wave becomes totally concentrated on the surfaces of two outgoing “resonance” cones from the source, while the electromagnetic component is more isotropically distributed. Thus, most of the electrostatic energy of waves of a given frequency is concentrated along resonance cone angles with respect to the magnetic field B_0 .

If the receiving antenna is placed so that the angle between the line from it to the source and B_o equals to the resonance cone angle, it will pick up large amplitude waves from the source. At other angles it does not pick up the electrostatic component of the wave, but only the much smaller amplitude electromagnetic component. As the wave frequency is changed the resonance cone angle also changes. For the wave frequencies of Figure 1 and the plasma conditions in Equation 1 the resonance cones do exist, so the electrostatic energy of the X-mode will be concentrated along resonance cones.

When the source emits a band of frequencies rather than a single frequency, then the receiving antenna will pick up the resonance cone frequency that is unique for the angle of observation, provided that resonance cone frequency lies in the band of emitted radiation. If the position of the receiving antenna is moved that will change the angle that the source-antenna line lies with respect to B_0 , so the frequency that it picks up will change.

The moving spacecraft changes the source-antenna angle with respect to B_0 and thus the source frequency received. For a decreasing resonance cone angle the frequency of the received wave also decreases. Before it first picks up frequencies from this band the spacecraft is at resonance cone angles that are appropriate for higher frequencies than are produced by the source. After it has received the lowest frequencies emitted the waveform cuts off.

The basic theory of the resonance cone emission in the X-mode for a homogeneous plasma and its application to the observed waves on Uranus is contained in Section 2. This shows that the resonance cone effect can produce the drop in the received frequency with time, and that the distance and angle of the source can be estimated. In Section 3 it is shown that inhomogeneous plasma effects can explain arches and sudden jumps in frequency. In Section 4 thermal effects are reviewed and shown to be able to explain multiple peaks in f seen at given time.

2. Resonance cone emission theory for homogeneous plasmas

Since the electrostatic wave component is much larger than the electromagnetic component along the resonance cone directions, the electrostatic approximation can be made for these waves. Thus the wave fields along these resonance cones involve the solution of Laplace's equation for the electrostatic wave potential modified by the dielectric tensor of the plasma.

Fig. 1 (color plot): f versus t for wave mode observed in the Uranian magnetosphere. The spacecraft was 31.1 Uranian radii from the surface of the planet.

In cylindrical coordinates (ρ, θ, z) , with the direction of B_0 chosen as the z-direction, the potential is (see e.g. Fisher and Gould, 1971)

$$\phi(\rho, z) = \frac{qe^{-2i\pi ft}}{4\pi\epsilon_0\sqrt{K_{\parallel}^2 K_{\perp}}\sqrt{(\rho^2/K_{\perp}) + (z^2/K_{\parallel})}} \quad (2)$$

where $\underline{k} = (k_x, 0, k_z)$ has been chosen (the x-direction is defined as the direction of the component of k perpendicular to B_0), and $K_{\perp}$ and $K_{\parallel}$ are perpendicular and parallel components of the following plasma dielectric tensor:

$$K = \begin{bmatrix} K & iK_H & 0 \\ -iK_H & K & 0 \\ 0 & 0 & K_{\parallel} \end{bmatrix} \quad (3)$$

They are given by:

$$\begin{aligned} K_{\perp} &= 1 - \frac{f_{pe}^2}{f(f + if_c) - f_{ce}^2} - \frac{f_{pi}^2}{f(f + if_c) - f_{ci}^2} \\ K_{\parallel} &= 1 - \frac{f_{pe}^2 + f_{pi}^2}{f(f + if_c)} \end{aligned} \quad (4)$$

where $f_c \ll f$ is a small collision frequency added to avoid unphysical infinities in Equation (2). For the following condition, ϕ becomes very large:

$$\tan \theta = \frac{\rho}{z} = Re \sqrt{\frac{-K_{\parallel}}{K_{\perp}}} \quad (5)$$

θ is the resonance cone angle with respect to $\underline{B}_0$. The surfaces of the resonance cones form two cones concentric with the magnetic field lines. Note that the resonance cones only occur when $Re(K_{\parallel}/K_{\perp}) < 0$.

The resonance cone angle in Equation (5) is (ignoring f_c):

$$\tan^2 \theta = -\frac{(f^2 - f_{uh}^2)f^2}{(f^2 - f_{pe}^2)(f^2 - f_{ce}^2)} \quad (6)$$

Resonance cones lie along the group velocity angle, which is perpendicular to the phase velocity angle for the X-mode (Stix, 1962).

From Equation (5) the resonance cones occur for in the following 3 ranges (branches):

$$\begin{aligned} f &< f_{ci} \\ f_{lh} &< f < f_{ce} \\ f_{pe} &< f < f_{uh} \end{aligned}$$

where $f_{uh}^2 = f_{pe}^2 + f_{ce}^2$ and $f_{lh} = f_{pi}/\sqrt{1 + f_{pe}^2/f_{ce}^2}$. For the conditions of the Figure 1 measurements on Uranus, the frequencies lies in the middle branch.

The spacecraft is moving from larger angles to smaller angles, as the decreasing frequency implies. The spacecraft is assumed to be moving along the magnetic field lines.

The position of the spacecraft will be parameterized as:

$$d(t) = d_0 - v_s t \quad (7)$$

where d_0 is indicated in Figure 2 as the distance the spacecraft travels begins receiving the waveform, v_s is the spacecraft velocity (equal to 20 km/sec for the Voyager 2 flyby of Uranus), and t the time since receiving begins. Simple trigonometric relationships for the geometric parameters in Figure 2 yield

$$d(t) + s = L / \tan[\theta(t)] \quad (8)$$

which has the conditions at the beginning ($t=0$) and end of the received frequency band ($t = d_0/v_s$):

$$s = L / \tan(\theta_1) \quad (9)$$

$$d_0 + s = L / \tan(\theta_2) \quad (10)$$

where θ_1 and θ_2 are measured in degrees. Solving for L :

$$L = \frac{d_0 \tan(\theta_1) \tan(\theta_2)}{\tan(\theta_1) - \tan(\theta_2)} \quad (11)$$

where d_0 is known, and the θ 's can be known from Equation (6) for f for the spacecraft at the 2 boundaries. “ s ” is also known from Equation (9), and $\theta(t)$ can then be found in Equation (8).

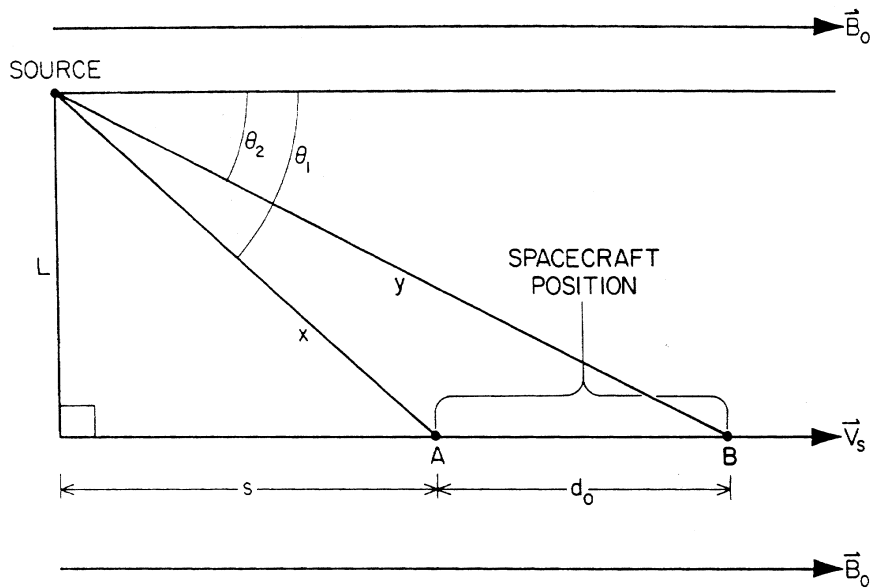


Fig. 2: Geometry for analyzing the spacecraft receiving the wave from its source, and the source-spacecraft angle with respect to the magnetic field changing.

An explicit expression for the $\theta(t)$ can be made by subtracting Equation (9) from (8) and making the linear small angle expansion of $\tan$. For the cases in Figure 1 the angle is less than about 10° , so the linear approximation is quite valid.

$$d_0 + s - v_s t = L/\theta(t) \quad (12)$$

$$\theta(t) = \beta - \alpha t \quad (13)$$

where:

$$\alpha = v_s/L \quad (14)$$

$$\beta = (d_0 + s)/L \quad (15)$$

In summary Equations (6) give the angle of observation and the distance of the source x or y in terms of the wave frequency observed and other plasma characteristics.

A time dependent function for the waveform will be predicted from the theory using Equations (6), and (13) – (15). In these cases $f^2 \ll f_{ce}^2$ so Equation (6) becomes:

$$f^2 = \frac{f_{pe}^2 f_{ce}^2 \tan^2\{L/(d_0 + s - v_s t)\} f^2(t)}{(f_{ce}^2 + f_{pe}^2) + f_{ce}^2 \tan^2\{L/(d_0 + s - v_s t)\}} \quad (16)$$

where L is known through Equation (11). This describes completely how the frequency changes as the position of the spacecraft changes for a homogeneous plasma. The predicted waveform $f(t)$ from this equation for the homogeneous theory is quite linear for the small angles of propagation from the source.

The homogeneous theory predicts the angle of observation with respect to B_0 , the distance of the source, and the variation in the waveform of the received frequencies as a function of time. Using Equations (8), (9), and (11) for the second waveform:

$$\begin{aligned} d_0 &= 40 \text{ km} & L &= 9.7 \text{ km} & s &= 45.4 \text{ km} \\ x &= 41.2 \text{ km} & y &= 95.9 \text{ km} \end{aligned} \quad (17)$$

Although the homogeneous theory may describe the waveforms in a rather gross way, they in fact have several characteristics that would require a more sophisticated model to predict. This includes a general arching over feature of waveform 2. The waveforms also have small abrupt drops and jumps in frequency, such as the one that first appears at about the 36.2 s. mark. These will be addressed in the inhomogeneous theory presented next section.

3. Resonance cone emission theory for inhomogeneous plasmas

Modifications will occur in the resonance cone spectrum predicted from homogeneous theory because of inhomogeneities in the plasma density and the background magnetic field. It will be assumed that these gradients are not sharp, and will just change the angle of the resonance cone. In general these are of 2 forms: almost monotonic gradients between the source and the field point of observation on the spacecraft, and hills or cavities in density or magnetic field between the source and point of observation.

The case of monotonic gradients between the source and point of observation will be dealt with first. These gradients just change the angle of the resonance cone as the local f_{pe} or f_{ce} is changed, which modifies the resonance cone angle to that appropriate for the new values of these frequencies. Thus inhomogeneities in this case only modify the homogeneous result of a linear spectrum of frequencies detected as a function of time if the density and magnetic field at the field point differs from the source. However, the modification of the resonance cone angle because of gradients continually changes with time because of the spacecraft motion. The final resonance cone angle detected by the spacecraft at each time is determined by the local density and magnetic field at the spacecraft.

The results of the previous can be generalized for this case by just replacing the magnetic field and plasma densities by their spatial dependence $B_0(x)$ and $n_0(x)$ where $x(t)$ is the local position of the spacecraft with time:

$$\tan^2 \theta(x) = -\frac{[f^2 - f_{uh}^2(x)]f^2}{[f^2 - f_{pe}^2(x)][f^2 - f_{ce}^2(x)]} \quad (18)$$

The values of $f_{pe}(x)$ and $f_{ce}(x)$ will be divided into their initial value at the source and the subsequent changes because of the gradient:

$$f_{pe}^2(x) = f_{pe0}^2 + \Delta f_{pe}^2 \quad f_{ce}(x) = f_{ce0} + \Delta f_{ce} \quad (19)$$

This separation will be used to expand $\tan^2 \theta(x)$. For a density gradient this expansion is:

$$\tan^2 \theta(x) = \tan^2 \theta_0 \frac{1 + \Delta f_{pe}^2 / (f_{uh}^2 - f^2)}{1 + \Delta f_{pe}^2 / (f_{pe0}^2 - f^2)} \quad (20)$$

For the case of the waves observed on Uranus this reduces to:

$$\tan^2 \theta(x) = \tan^2 \theta_0 \frac{1 + 0.77(\Delta n/n_0)}{1 + 1.01(\Delta n/n_0)} \quad (21)$$

where n_0 and Δn are the initial density and the change in density due to the gradient.

For a magnetic field gradient the expansion of $\tan^2 \theta(x)$ is:

$$\tan^2 \theta(x) = \tan^2 \theta_0 \frac{1 + [(2f_{ce} + \Delta f_{ce})\Delta f_{ce} / (f_{uh}^2 - f^2)]}{1 + [(2f_{ce} + \Delta f_{ce})\Delta f_{ce} / (f_{ce}^2 - f^2)]} \quad (22)$$

For the case of the waves observed on Uranus this reduces to

$$\tan^2 \theta(x) = \tan^2 \theta_0 \frac{1 + 0.23(2 + \Delta B/B_0)(\Delta B/B_0)}{1 + 1.03(2 + \Delta B/B_0)(\Delta B/B_0)} \quad (23)$$

These results for a linear density or magnetic field gradient have several consequences. Figure 3 is a graph of frequency detected versus time for an increasing magnetic field. The graph shows that a small arching over in the frequency can be achieved for an increasing magnetic field producing a 50% changes in the magnetic field over the period of the waveform. Almost the same result is achieved for a 50% reduction in the magnetic field. An increase in density has a similar effect, although the effect of the inhomogeneity is smaller, so that it requires over a 100% density change to produce the same effect. Thus linear increases in density or magnetic field can produce local arches, but apparently cannot produce the large arch observed in waveform 2.

The arch in waveform 2 could be the result of an unusual density gradient, in which the negative slope is growing at the position of the detection on the spacecraft. In Figure 4 we have graphed $f(t)$ for an assumed nonlinear magnetic field profile of this nature. Such a profile might result when the spacecraft is just beginning to enter a density or magnetic field cavity where a gentle slope downward is changing to a steeper slope downward. This graph shows that such a profile *can* explain the arching over characteristic that is observed in waveform 2.

The second form of density gradient is that in which a cavity or hill in the density or magnetic field occurs between the source and the point of observation. This case cannot be subject to the same analytical treatment as the first case, but the effects can be understood qualitatively from Figure 5. The hill or cavity produces a change in resonance cone angle, which then reverts back to its initial value. However, that change effects when the spacecraft sees that particular angle. Figure 5 shows that it modifies the linear drop of f with time in the homogeneous case to periods when the rate of drop is slower and faster than the linear case.

This second form of density gradient can explain several effects seen in the data, particularly for waveform 1. There are periods when the waveform changes very gradually followed by sudden drops in frequency and regions where the drop in frequency becomes more steep. Local cavities can produce several of the arches and quirks in waveform 1.

In addition, this form of inhomogeneity provides a second possible explanation of the arching over in frequency observed in waveform 2. This could be produced by a large cavity, in which the spacecraft was just started to detect resonance cones that had passed the edge of the cavity. As the spacecraft moved, it picked up resonance cones that were undergoing continually greater displacement produced by the cavity. This caused the rate of fall of frequency to become increasingly greater, producing the overarch observed in

waveform 2.

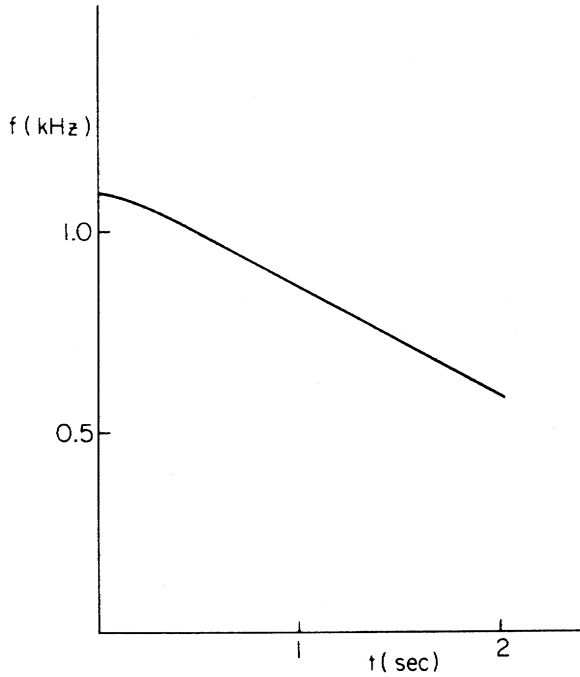


Fig. 3: Arch in the waveform produced from an inhomogeneous linearly increasing magnetic field. At the end of the waveform it is 50% larger than at the beginning.

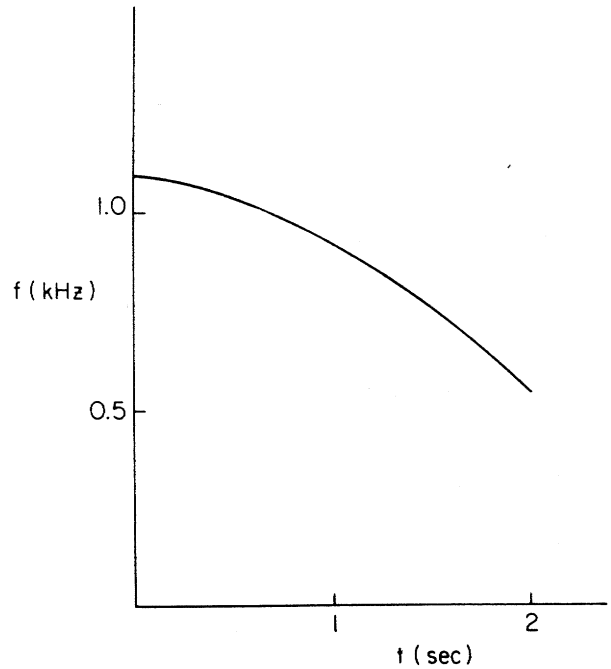


Fig. 4: Arch in the waveform produced from an inhomogeneous nonlinearly decreasing magnetic field. The slope is growing linearly, and at the end of the waveform B_0 is 0.36 its value at the beginning.

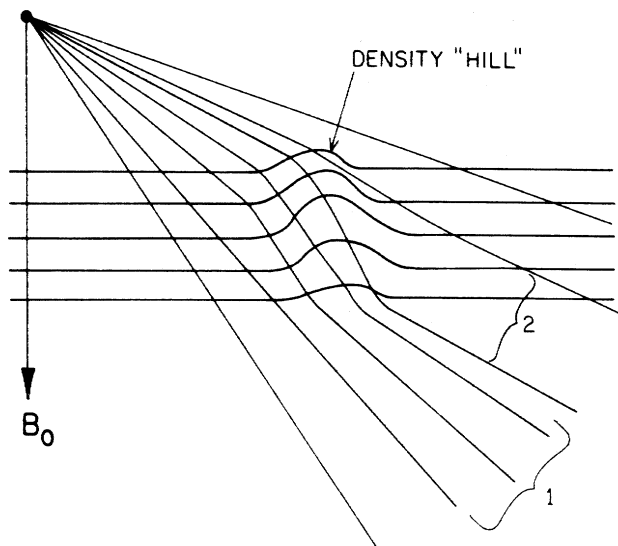


Fig. 5: Modification of the resonance cones received at the spacecraft by a density or magnetic field hill between the source and the field point. This causes the spacecraft to detect a region of slow change in resonance cone angle, following by a region of rapid change in resonance cone angle. For a density or magnetic field cavity the regions would be reversed.

4. Thermal effects on the resonance cones

The thermal effects on resonance cones of the extraordinary mode are well-understood (Fisher and Gould, 1971; Kuehl, 1973). This thermal component of the wave produces an interference structure of the resonance cone wave field observed near the resonance cone angle. The maximum in the wave field gets shifted slightly from the resonance cone direction, and secondary maxima in the field also appear (see Figure 6).

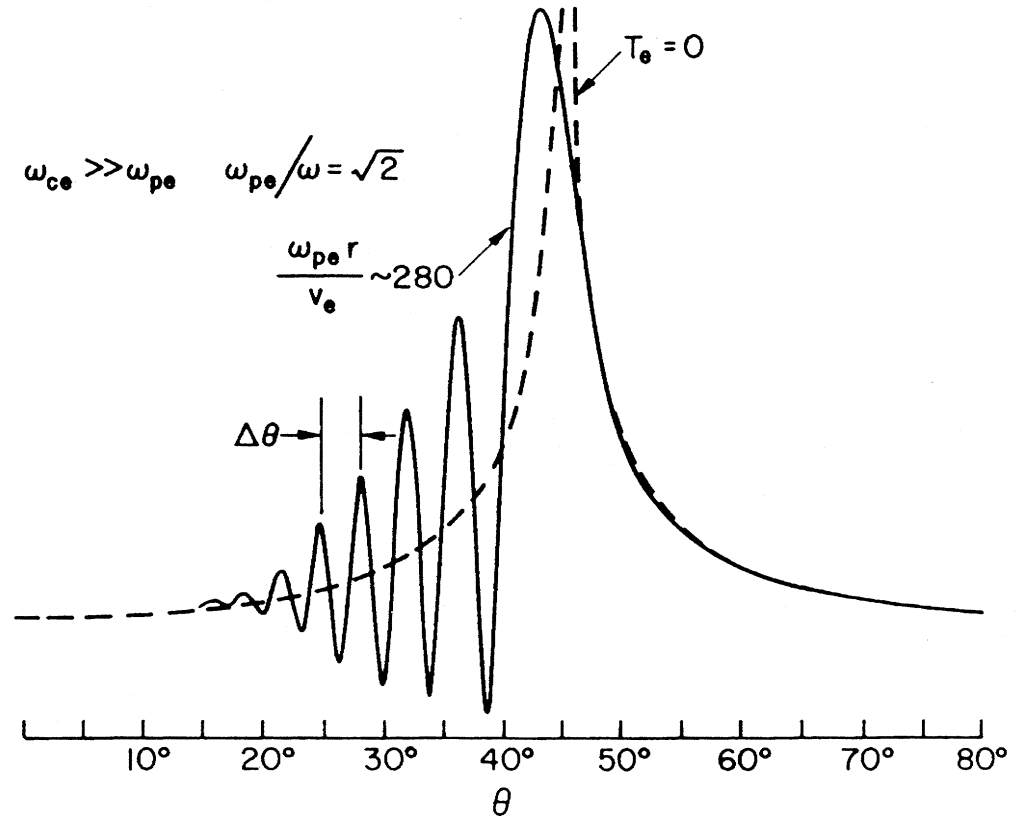


Fig. 6: Cross-section of the electrostatic amplitude of the field of the wave received in the resonance cone direction when thermal effects on the resonance cone are taken into account (after Fisher and Gould, 1971).

These thermal effects will have an important consequence for the data: the wave picked up at a given time should have more than one closely-spaced peak in frequency. The reason is, a frequency maximum is picked up for the appropriate resonance cone at the frequency, and secondary maxima are picked up for nearby frequencies. This causes the spectrum detected at a given time to contain multiple peaks that are closely spaced in frequency.

This is what is observed quite often in the data in Figure 1. In general the waveform at any given time is spread out over 1 Hertz in frequency, and quite often exhibits 2 or 3 peaks at slightly different frequencies at a given time. The occurrence of these closely spaced peaks is well-explained as the thermal effects of resonance cones, as is the general width and structure to the frequency picked up at a given time.

5. Summary

The observed waveforms can be well-explained as bands of X-mode waves on the resonance cone. The predictions made from this theory for the general shapes of the waveforms in Figure 1 can explain almost all the features of the observed waveforms. The homogeneous theory allows the data to be used to determine the angle at which the source lies with respect to the spacecraft, and how far away it is. These predictions can be used for future useful tests of the theory by the use of independent experimental means of measuring that distance and the angles.

Two models of plasma inhomogeneities were introduced that can explain the arching over nature of the second waveform. In addition, certain fine structure features such as the fine-scale modulation of the received frequencies and the abrupt small drop in the frequency observed for one waveform can be readily explained by inhomogeneities in the plasma. Finally, the multiple peaks in frequency for a given time can be easily produced by thermal effects on the resonance cone.

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