

DIRECTION FINDING OF ELECTROMAGNETIC WAVES: APPLICATION TO PLANETARY RADIO EMISSION

H. O. Rucker*, H. P. Ladreiter*, W. Macher* and G. K. F. Rabl*

Abstract

One major goal in the physics of planetary radio emission is the determination of the source location in the magnetosphere, its extension, and the state of polarization of the emission. A simple approach to determine the source location is to study the visibility of possible source regions from the position of the spacecraft during periods of observed radio emission. If there is a considerable plasma density between the radio source and the spacecraft, ray-tracing might be necessary to accomplish the correct source location. However, a more powerful approach to simultaneously determine the source location and the polarization of the radio emission is the method of direction finding. This can be performed by using two antennas on a spinning spacecraft or three antennas on a three-axes stabilized spacecraft. It will be shown how the method of direction finding can be applied to a spacecraft with two short orthogonal antennas which is (a) rotating and (b) three-axes stabilized. In the former case, the Stokes parameters (characterizing the wave polarization) and the source direction can be derived simultaneously whereas in the latter case, some prior knowledge of the source direction or polarization is necessary.

1 Introduction

The determination of a radio source location within a planetary magnetosphere is possible by two indirect methods (A and B) and by one direct method (C):

A) Determination of the "Radio Horizon of Visibility"

Following the widely accepted theory on the generation of planetary radio emission, the so-called "Cyclotron Maser Instability" [Wu and Lee, 1979], the emitted radio frequency is close to the electron cyclotron frequency, $f_{wave} \simeq f_c$. Then the radio horizon of visibility is the visible area from the point of observation (i.e. the spacecraft) on the $f_c = \text{const.}$ -contour (Fig. 1a) which is defined by the magnetic field configuration of the respective planet.

*Space Research Institute, Austrian Academy of Sciences, Halbärthgasse 1, 8010 Graz, Austria

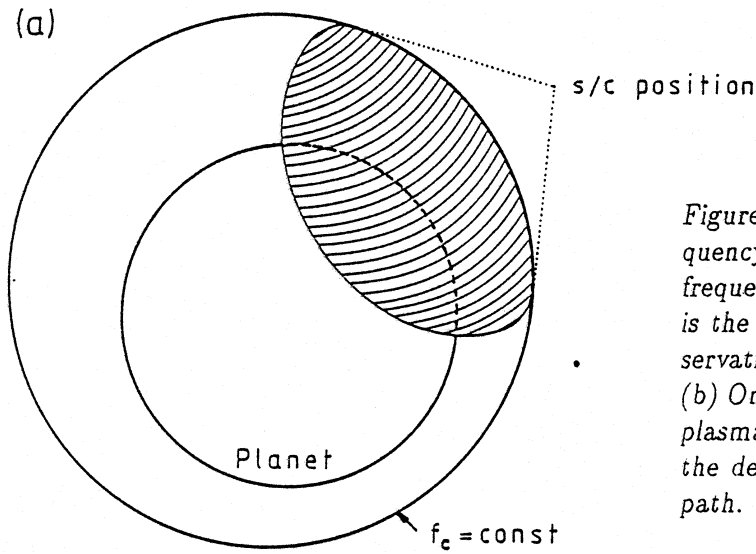
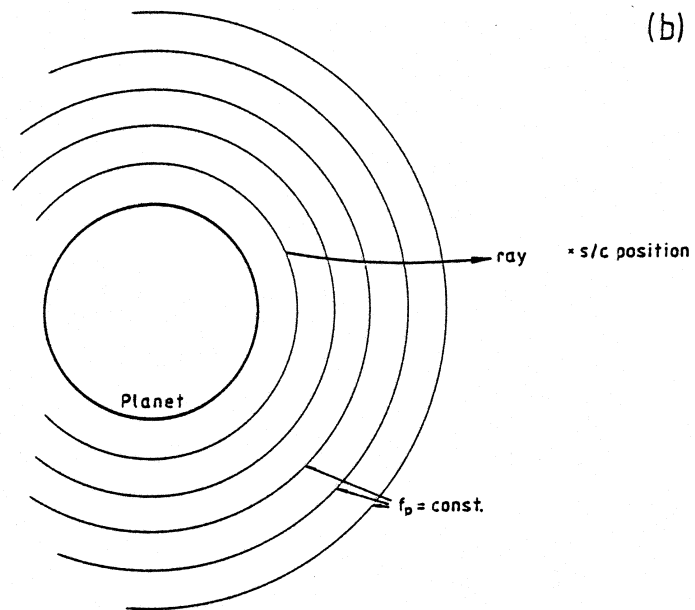


Figure 1: (a) Since the emitted radio frequency is close to the electron cyclotron frequency, the radio horizon of visibility is the visible area from the point of observation on the $f_c = \text{const.}$ -contour. (b) On the basis of known magnetic and plasma environment ray-tracing enables the determination of a ray propagation path.



B) Ray-Tracing

Taking into account the magnetic field structure and the plasma density in the region between the radio source location and the observer, a radio mode can be released and traced through any defined magnetoplasma. Thus, ray-tracing enables the determination of a ray propagation path and yields important details about a wave's origin, mode, and polarization (Fig. 1b).

C) Direction Finding

Contrary to methods (A) and (B) the so-called direction finding is a direct method in order to determine the wave's origin and polarization. An unambiguous determination of the wave parameters and source direction is possible by using three linear independent antennas or by rotation of two linear independent antennas (which subsequently leads to uncertainties and limitations). In the following this latter case is outlined in some detail.

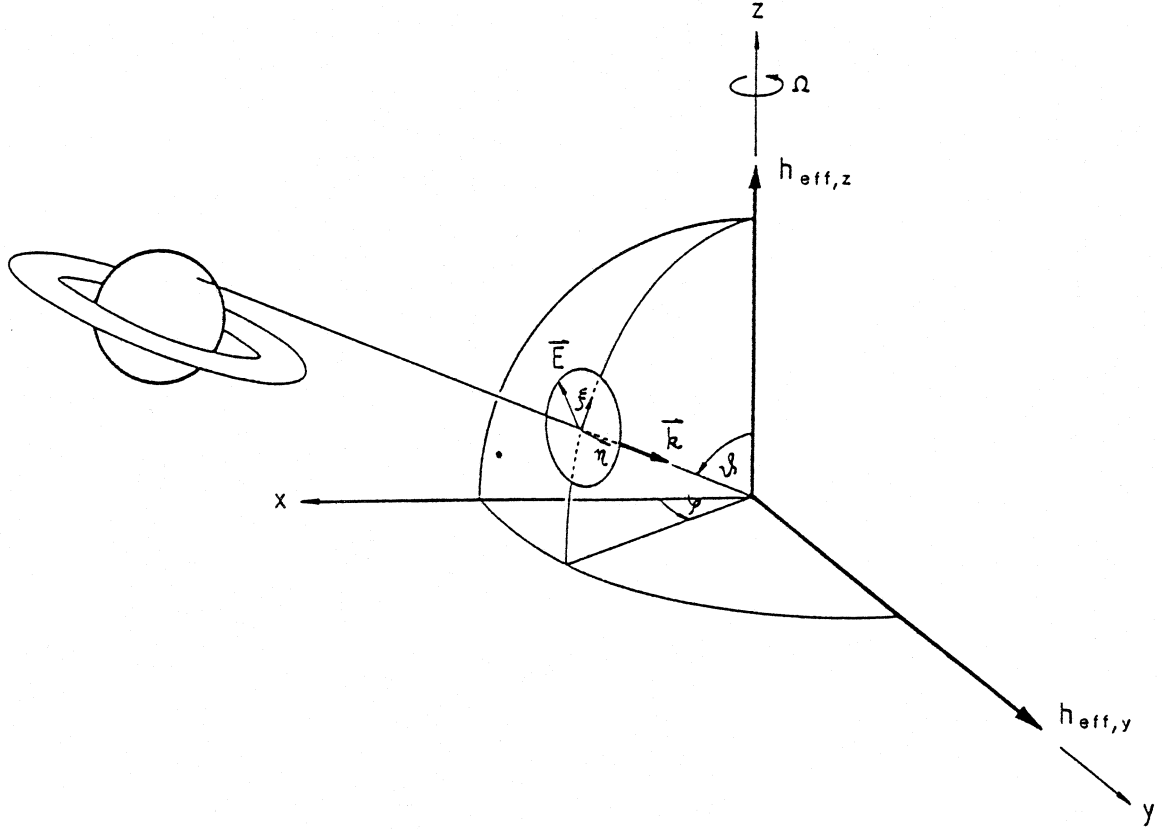


Figure 2: In the case of a radio wavelength λ much greater than the physical length of the antennas and when ignoring the influence of the spacecraft body on the antennas the effective height h_e is approximately half the physical antenna length and $\vec{h}_e$ is almost parallel to the respective antenna. In the antenna coordinate system, as shown, the direction of the incident radio wave is defined by the angles θ and φ .

2 Formalism of direction finding

Using two antennas for radio wave direction finding the spacecraft must be rotated to allow the source direction and the wave polarization to be simultaneously derived, with the spinning plane out of the antenna plane. In the course of spacecraft rotation a modulation of the received voltage is observed. The response of the receiving antenna is given by

$$V = \vec{h}_e \cdot \vec{E}, \quad (2.1)$$

where $\vec{h}_e$ is the effective height of the antenna and $\vec{E}$ is the incident electric field of the radio wave. The effective height $\vec{h}_e$ is a function of the wavelength, source direction, physical length of the antenna and its geometry with respect to the spacecraft [Ortega-Molina and Daigne, 1984]. Thus, the effective height $\vec{h}_e$ characterizes the antenna configuration with respect to the incident wave.

In the case of an antenna size comparable to those on Voyager, Ulysses or on the planned Cassini spacecraft the wavelengths of a non-thermal radio component in the kilometric wavelength range (e.g. AKR, KOM, SKR) is very long compared to the dimensions of the antenna system. Therefore the effective heights are approximately one half of the physical length of the antennas, and $\vec{h}_e$ is almost parallel to the respective antennas (Figure 2).

However, the spacecraft body may considerably influence the receiving properties of the mounted antennas if the spacecraft dimensions are comparable to the antenna length. This was the case for both Voyagers [Ortega–Molina and Daigne, 1984]. In that case the effective height $\vec{h}_e$ of the antenna–spacecraft configuration must be calculated as was done by Sawyer et al. [1991] by using the method of moments. When knowing the effective height of the antenna system the direction of the incident radio wave $\vec{E} = \vec{E}_0 \exp[i(\vec{k} \cdot \vec{r} - \omega t)]$ (with the orthogonal components $E_{0,\xi}$ and $E_{0,\eta}$) is defined by the angles θ and φ in the antenna coordinate system x, y, and z (Figure 2).

The response of the antennas due to the incident radio waves results in 4 independent measured values ($\langle \rangle$ denotes the time average).

$\langle V_y V_y^* \rangle$ is the voltage squared, received by the y–antenna.

$\langle V_z V_z^* \rangle$ is the voltage squared, received by the z–antenna.

$\text{Re} \langle V_y V_z^* \rangle$ is the real part and

$\text{Im} \langle V_y V_z^* \rangle$ the imaginary part of cross correlation of measured voltages.

Following investigations by Lecacheux [1978], Fainberg [1979] and Ladreiter et al.[1991] the time–averaged quantities are

$$\begin{aligned} \langle V_y V_y^* \rangle &= \frac{1}{4} h_{e,y} h_{e,y}^* [\langle E_{0,\xi}^2 \rangle \cos^2 \theta + \langle E_{0,\eta}^2 \rangle + \\ &\quad \cos(2\Omega - 2\varphi) (\langle E_{0,\xi}^2 \rangle \cos^2 \theta - \langle E_{0,\eta}^2 \rangle) - \\ &\quad \sin(2\Omega - 2\varphi) \langle E_{0,\xi} E_{0,\eta} \cos \delta \rangle \cos \theta], \end{aligned} \quad (2.2)$$

$$\langle V_z V_z^* \rangle = \frac{1}{2} h_{e,z} h_{e,z}^* \langle E_{0,\xi}^2 \rangle \sin^2 \theta, \quad (2.3)$$

$$\begin{aligned} \Re \langle V_y V_z^* \rangle &= \frac{1}{2} h_{e,y} h_{e,z}^* [\cos(\Omega - \varphi) \langle E_{0,\xi}^2 \rangle \cos \theta \sin \theta + \\ &\quad \sin(\Omega - \varphi) \langle E_{0,\xi} E_{0,\eta} \cos \delta \rangle \sin \theta], \end{aligned} \quad (2.4)$$

$$\Im \langle V_y V_z^* \rangle = \frac{1}{2} h_{e,y} h_{e,z}^* \sin(\Omega - \varphi) \langle E_{0,\xi} E_{0,\eta} \sin \delta \rangle \sin \theta. \quad (2.5)$$

Introducing the normalized Stokes parameters (with Z as the antenna impedance and S as the Poynting vector)

$$\frac{s_0}{2Z} = S = \frac{\langle E_{0,\xi}^2 + E_{0,\eta}^2 \rangle}{2Z}, \quad (2.6)$$

$$\frac{s_1}{s_0} = q = \frac{\langle E_{0,\xi}^2 - E_{0,\eta}^2 \rangle}{\langle E_{0,\xi}^2 + E_{0,\eta}^2 \rangle}, \quad (2.7)$$

$$\frac{s_2}{s_0} = u = 2 \frac{\langle E_{0,\xi} E_{0,\eta} \cos \delta \rangle}{\langle E_{0,\xi}^2 + E_{0,\eta}^2 \rangle}, \quad (2.8)$$

$$\frac{s_3}{s_0} = v = 2 \frac{\langle E_{0,\xi} E_{0,\eta} \sin \delta \rangle}{\langle E_{0,\xi}^2 + E_{0,\eta}^2 \rangle}, \quad (2.9)$$

we obtain for Equations (2.2–2.5)

$$\langle V_y V_y^* \rangle = \frac{1}{4} S Z h_{e,y} h_{e,y}^* [1 + \cos^2 \theta - q \sin^2 \theta + \cos(2\Omega - 2\varphi) (q \cos^2 \theta +$$

$$q - \sin^2 \theta) - \sin(2\Omega - 2\varphi)2u \cos \theta], \quad (2.10)$$

$$\langle V_z V_z^* \rangle = \frac{1}{2} S Z h_{e,z} h_{e,z}^* (q + 1) \sin^2 \theta, \quad (2.11)$$

$$\Re \langle V_y V_z^* \rangle = \frac{1}{4} S Z h_{e,y} h_{e,z}^* [-\cos(\Omega - \varphi)(1 + q) \sin 2\theta + \sin(\Omega - \varphi)2u \sin \theta], \quad (2.12)$$

$$\Im \langle V_y V_z^* \rangle = \frac{1}{4} S Z h_{e,y} h_{e,z}^* \sin(\Omega - \varphi)2v \sin \theta. \quad (2.13)$$

Equations (2.10–2.13) relate the measured quantities $\langle V_y V_y^* \rangle$, $\langle V_z V_z^* \rangle$, and $\langle V_y V_z^* \rangle$ to the 4 Stokes parameters and the source direction (azimuth φ and polar distance θ). Four equations for six unknowns require at least two independent measurements of the values $\langle V_y V_y^* \rangle$, $\langle V_z V_z^* \rangle$, and $\langle V_y V_z^* \rangle$ with different antenna–source geometries (varying Ω) in order to yield an unambiguous determination of the Stokes parameters and the source direction.

3 Application of direction finding

3.1 Rotating spacecraft

By the assumption of spacecraft rotation around the z-axis the spin frequency f will cause a modulation at frequencies f and $2f$ on the received radio signals. As can be seen by Equation (2.10) the measured voltage $\langle V_y V_y^* \rangle$ is modulated by twice the spin frequency $f = \Omega/t$. The real and imaginary parts of cross correlation of the measured voltages (Equations 2.12 and 2.13) are modulated by f . The voltage received by the z-antenna $\langle V_z V_z^* \rangle$ (Equation 2.11) shows no modulation, since this antenna is assumed to be parallel to the spin axis.

In order to obtain the relevant information on the wave characteristics (the Stokes parameters S , q , u , v) and the source direction (θ , φ) we have to Fourier analyze Equations (2.10–2.13). Since Z and $\vec{h}_{e,y}$, $\vec{h}_{e,z}^*$ are known properties of the antenna configuration, these Equations (2.10–2.13) can be written as

$$Y1 = \frac{4 \langle V_y V_y^* \rangle}{Z h_{e,y} h_{e,y}^*} = S [C_y(q, \theta) + B_y(q, \theta) \cos(2ft - 2\varphi) + A_y(u, \theta) \sin(2ft - 2\varphi)], \quad (3.1)$$

$$Z1 = \frac{4 \langle V_z V_z^* \rangle}{Z h_{e,z} h_{e,z}^*} = S C_z(q, \theta), \quad (3.2)$$

$$R1 = \frac{4 \Re \langle V_y V_z^* \rangle}{Z h_{e,y} h_{e,z}^*} = S [B_r(q, \theta) \cos(ft - \varphi) + A_r(u, \theta) \sin(ft - \varphi)], \quad (3.3)$$

$$I1 = \frac{4 \Im \langle V_y V_z^* \rangle}{Z h_{e,y} h_{e,z}^*} = S [A_i(v, \theta) \sin(ft - \varphi)]. \quad (3.4)$$

The values C_y , C_z , A_y , B_y , A_r , B_r , and A_i are functions of the polarization of the signal and the source direction. The azimuth φ of the source direction is trivially derived by setting

$ft_v = \varphi$ at times t_v when $I1$ vanishes. In the case of a spinning spacecraft the relevant quantities can be derived by Fourier analysis from the modulation of the measured values $Y1(t_i)$, $Z1(t_i)$, $R1(t_i)$, and $I1(t_i)$ at discrete time points t_i

$$C_y = \frac{1}{N} \sum_{i=1}^N Y1(t_i), \quad (3.5)$$

$$C_z = \frac{1}{N} \sum_{i=1}^N Z1(t_i), \quad (3.6)$$

$$A_y = \frac{2}{N} \sum_{i=1}^N Y1(t_i) \sin(2ft_i - 2\varphi), \quad (3.7)$$

$$B_y = \frac{2}{N} \sum_{i=1}^N Y1(t_i) \cos(2ft_i - 2\varphi), \quad (3.8)$$

$$A_r = \frac{2}{N} \sum_{i=1}^N R1(t_i) \sin(ft_i - \varphi), \quad (3.9)$$

$$B_r = \frac{2}{N} \sum_{i=1}^N R1(t_i) \cos(ft_i - \varphi), \quad (3.10)$$

$$A_i = \frac{2}{N} \sum_{i=1}^N I1(t_i) \sin(ft_i - \varphi). \quad (3.11)$$

The required values S, q, u, v , and θ (φ is known from Equation (3.4)) can easily be derived from the values $C_y, C_z, A_y, B_y, A_r, B_r$, and A_i . As an example we demonstrate in Figure 3 the expected modulation of a test signal characterized by $S = S_0, q = 0.6, u = 0.0, v = -0.8, \theta = 20^\circ, \varphi = 0^\circ$ during one spacecraft rotation. Signal measurements during several spin periods then enable the determination of the wave polarization and the source direction with great accuracy as shown by Fainberg [1979]. The above described analysis applies to the case of two antennas on a spinning spacecraft. A three axes stabilized spacecraft with only two antennas must perform roll maneuvers to simulate the spinning. Thus direction finding is only possible during these particular periods of time of roll maneuvers.

3.2 Three-axes stabilized spacecraft

Without rotation of the spacecraft we see that one simple set of the 4 voltages as defined in Equations (2.10 – 2.13) is not sufficient to solve for the 6 unknowns ($\theta, \varphi, S, q, u, v$). Thus, prior assumptions must be made either regarding the source position (θ, φ) or the polarization state of the wave to reduce the number of unknowns to four. From Voyager observations it is known (see Ortega-Molina and Lecacheux, 1991) that planetary radio emissions in the kilometric and hectometric wavelength range are purely circularly polarized. Thus, by setting $q = u = 0$ (q and u define the degree of linear polarization) the number of unknowns in Equations (2.10 – 2.13) is reduced to four.

We have to point out that a number of restrictions caused by the involved physics will deteriorate the results. The assumption of a radio wave emitting source with infinitesimal

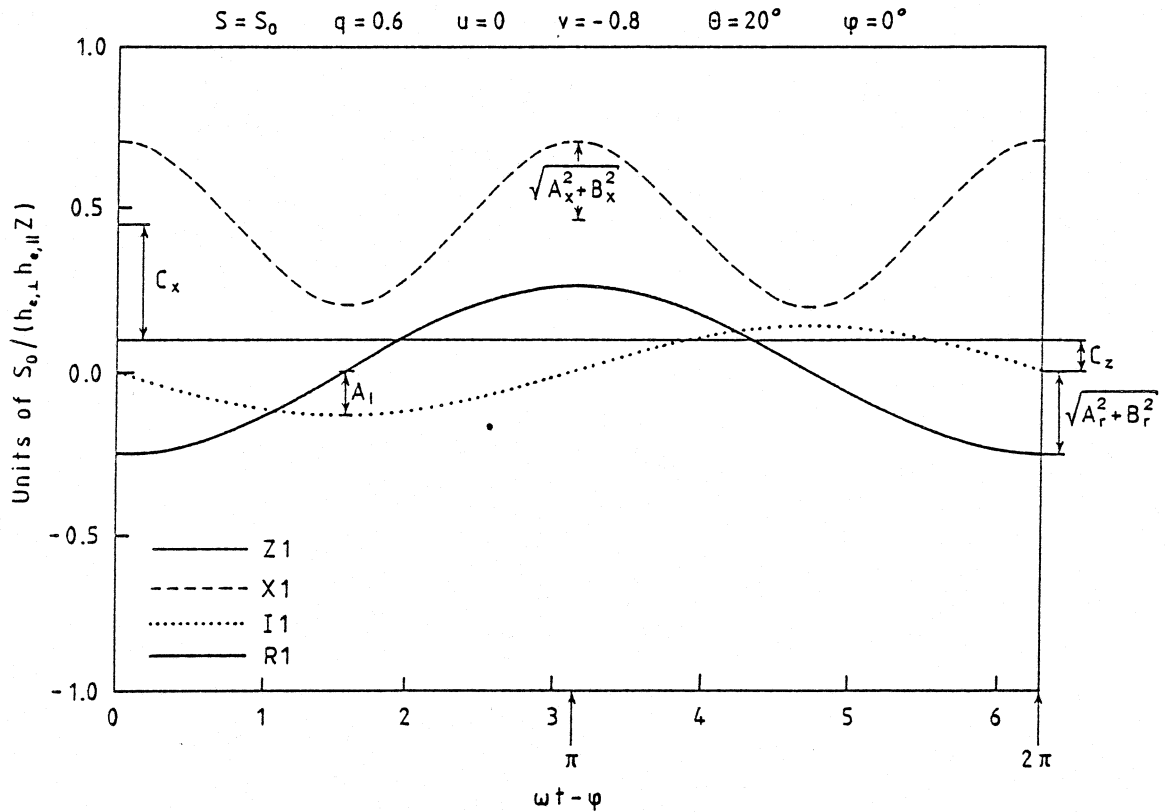


Figure 3: From a defined radio wave signal four independent quantities can be measured by two antennas.

extent as well as the stationarity of emission during the observation is an idealizing of reality. Additional sources of errors are the varying antenna-source geometry and the almost unknown influence of the spacecraft body on the directivity diagram of the antenna system.

Further investigation with regard to these restrictions will be done. But it certainly remains advisable to limit the performance of direction finding to favourable periods of time when the errors arising from these restrictions are expected to be a minimum.

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