

NONLINEAR GENERATION OF RADIATION IN PLANETARY MAGNETOSPHERES

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Abstract

A novel nonlinear mechanism for generating narrow band electromagnetic radiation in a nonuniform magnetoplasma is analyzed. Specifically, it is shown that a large amplitude electrostatic upper-hybrid wave can parametrically excite magnetic-field-aligned left-handed circularly polarized radiation as well as almost perpendicularly propagating low-frequency electrostatic modes. The nonlinear dispersion relation, the growth rate, and the threshold of the parametric instabilities are derived. It is suggested that the present nonlinear mechanism can be responsible for the narrowband radiation which is frequently observed in the planetary magnetospheres.

1 Introduction

The plasma wave spectra from the five planetary magnetospheres (viz. Earth, Jupiter, Saturn, Uranus, and Neptune) contain signatures of coherent electrostatic and electromagnetic waves [e.g. Etcheto et al., 1982; Gough, 1982; Barbosa, 1982; Gurnett et al., 1986]. Specifically, observations indicate that nonthermal electrostatic waves and continuum radiation coexist near and just outside the equatorial plasmopause in the terrestrial magnetosphere. Thus, there is a possibility that the continuum radiation can result from conversion of an electrostatic upper-hybrid (UH) wave, which is excited due to free energy available in electron beams, anisotropic electron velocity distribution functions, etc.. Four mechanisms have been proposed for the radiation from the planetary magnetospheres. These are: (i) linear mode conversion in a density gradient [Jones, 1976], (ii) nonlinear coalescence between the UH wave and some low-frequency electrostatic wave in a uniform plasma [Melrose, 1981], (iii) decay of the UH wave into a high-frequency (HF) left-handed circularly polarized (LHCP) electromagnetic (EM) wave along the geomagnetic field lines, and a low-frequency (LF) electrostatic (ES) mode (preferentially lower-hybrid) [Murtaza and Shukla, 1984], and (iv) radiation from collapsing upper-hybrid solitons [Christiansen

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et al., 1984]. An evaluation of the various processes has been made by R  nnmark [1985, 1989].

However, the planetary magnetospheres contain an equilibrium density gradient, which can influence the propagation of the waves. It is therefore of practical interest [Stenflo and Shukla, 1991] to generalize the theory for the parametric decay instability [UH $\rightarrow$ radiation + low-frequency ES waves] including the effect of the plasma inhomogeneity on the low-frequency longitudinal waves. This is the purpose of the present paper.

2 Theory

We consider the nonlinear interaction of a large amplitude upper-hybrid (UH) wave with a fully ionized nonuniform plasma in an external magnetic field $\mathbf{B}_0$, aligned with the z axis. The equilibrium density gradient is along the x axis; i. e., $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ and $n_{j0} = n_0(x)$, where B_0 is the strength of the constant magnetic field, n_0 is the unperturbed particle number density, and $j = e(i)$ stands for electrons (ions). We suppose that a finite UH electrostatic wave is excited by electron beams in our planetary magnetosphere. The resonant nonlinear interaction of a monochromatic UH pump wave $(\omega_0, \mathbf{k}_0)$ with a low-frequency electrostatic test wave $(\omega_l, \mathbf{k}_l)$ gives rise to a nonlinear current which becomes the source for the radiation $(\omega_t, \mathbf{k}_t)$. For illustrative purposes, the latter is here assumed to be left-handed circularly polarized (LHCP) propagating along the guide field $B_0 \hat{\mathbf{z}}$. The frequency matching condition [Murtaza and Shukla, 1984] is $\omega_0 = \omega_t + \omega_l$. As $\omega_0/k_0 c \ll 1$ for our electrostatic wave and as $\omega_t/k_t c$ can be of order unity, and as $\omega_l \ll \omega_0$, it is clear that $|\mathbf{k}_t| \ll |\mathbf{k}_0|$. Thus, in the following, we shall denote both $\mathbf{k}_0$ and $\mathbf{k}_l$ by the same vector $\mathbf{k}$. Furthermore, c is the speed of light.

The nonlinear interaction between the two longitudinal waves generates a LHCP transverse wave, whose electric field E_{t-} is given by

$$[k_t^2 c^2 - \omega_t^2 + \omega_{pe}^2 \omega_t / (\omega_t - \omega_{ce})] E_{t-} \approx (q_e \omega_t / \epsilon_0) [n_0 (\mathbf{v}_{el}^* \cdot \nabla v_{e0-} + \mathbf{v}_{e0} \cdot \nabla v_{el-}^*) / (\omega_t - \omega_{ce}) + i n_{el}^* v_{e0-}], \quad (2.1)$$

where $E_{t-} = E_{tx} - i E_{ty}$, $v_{e-} = v_{ex} - i v_{ey}$, $\omega_{pe}^2 = n_0 q_e^2 / m_e \epsilon_0$, $\omega_{ce} = q_e B_0 / m_e$, q_e / m_e is the electron charge to mass ratio, n_{el} is the electron number density perturbation, $\mathbf{v}_e$ is the electron fluid velocity, and the asterisk denotes the complex conjugate. Noting that $\mathbf{v}_e \cdot \nabla \mathbf{v}_e$ is approximately equal to $-\mathbf{v}_e \times (\nabla \times \mathbf{v}_e)$ (because $\mathbf{k}_t$ is small), and that for $\omega_l \ll |\omega_{ce}|$ and $k_\perp^2 \rho_e^2 \ll 1$,

$$n_{el} \approx -n_0 \frac{q_e}{m_e} \frac{\omega_l^2}{(\omega_l^2 - k_\perp^2 v_{te}^2)} \left[k_\perp^2 + \frac{k_y |\omega_{ce}|}{L \omega_l} - \omega_{ce}^2 \frac{k_z^2}{\omega_l^2} \right] \frac{\phi_l}{\omega_{ce}^2}, \quad (2.2)$$

where $L = -n_0 / d_x n_0$, $\rho_e = v_{te} / |\omega_{ce}|$ is the electron Larmor radius, $v_{te} = (T_e / m_e)^{1/2}$ is the electron thermal velocity, T_e is the electron temperature, and ϕ_l is the potential of the low-frequency wave, we write (2.1) in the form

$$D_t E_{t-} = -i (q_e / m_e) [\omega_0 \omega_{pe}^2 k_- / \omega_l^2 (\omega_0 - \omega_{ce})] C(\omega_l, \mathbf{k}) \phi_l^* \phi_0, \quad (2.3)$$

where $D_t = k_t^2 c^2 - \omega_t^2 + \omega_{pe}^2 \omega_t / (\omega_t - \omega_{ce})$, $k_- = k_x - i k_y$, and $C(\omega_l, \mathbf{k}) = [k_\perp^2 \omega_l^2 / \omega_{ce}^2 (1 - \omega_{ce}^2 / \omega_0^2) - (k_y \omega_l / L \omega_{ce}) - k_z^2] \omega_l^2 / (\omega_l^2 - k_z^2 v_{te}^2)$. In deriving (2.3), we have made use of the electron fluid velocity in the field of the UH pump, i.e. $v_{e0-} = q_e k_- \phi_0 / m_e \omega_0 (1 - \omega_{ce} / \omega_0)$, where ϕ_0 is the potential of the UH pump.

The interaction between the UH pump and the radiation reinforces the low-frequency ($\omega_{ci} \ll \omega_l \ll |\omega_{ce}|$, where $\omega_{ci} = q_i B_0 / m_i$ is the ion gyrofrequency and q_i / m_i is the ion charge to mass ratio) electrostatic modes. The perpendicular and parallel (to $\mathbf{B}_0$) components of the low-frequency part of the electron fluid velocity are given by

$$\begin{aligned} \mathbf{v}_{el\perp} \approx & (q_e / m_e \omega_{ce}) \hat{\mathbf{z}} \times \nabla \phi_l + \hat{\mathbf{z}} \times \mathbf{v}_{et}^* \cdot \nabla v_{e0\perp} / \omega_{ce} \\ & - (\partial_t / \omega_{ce}^2) [(q_e / m_e) \nabla_\perp \phi_l + \mathbf{v}_{et}^* \cdot \nabla \mathbf{v}_{e0\perp}], \end{aligned} \quad (2.4)$$

and

$$\partial_t v_{elz} = -(q_e / m_e) \partial_z \phi_l - (v_{te}^2 / n_0) \partial_z n_{el} + \mathbf{v}_{et}^* \cdot \nabla v_{e0z}, \quad (2.5)$$

where $v_{et}^* = -i q_e E_{t-}^* / m_e (\omega_t - \omega_{ce})$.

Substituting (2.4) and (2.5) into the electron continuity equation

$$\partial_t n_{el} + n_0 \nabla \cdot \mathbf{v}_{el} + (q_e / m_e \omega_{ce}) \hat{\mathbf{z}} \times \nabla \phi_l \cdot \nabla n_0 \approx -\mathbf{v}_{et}^* \cdot \nabla n_{e0}, \quad (2.6)$$

eliminating n_{el} by means of the Poisson's equation

$$n_{el} = n_{il} - \frac{\epsilon_0}{q_e} \nabla^2 \phi, \quad (2.7)$$

making use of the ion number density perturbation n_{il} , and neglecting the $k_y \omega_l / L \omega_{ce}$ term in C , we obtain after Fourier transformation

$$\omega_l^2 k^2 \epsilon_l \phi_l \approx i [q_e \omega_{pe}^2 / m_e \omega_0 (\omega_0 - \omega_{ce})] C(\omega_l, \mathbf{k}) k_- E_{t-}^* \phi_0, \quad (2.8)$$

where $\epsilon_l = 1 + \chi_e + \chi_i$ is the dielectric constant. The electron and ion susceptibilities [Stenflo, 1981] are given by,

$$\chi_e \approx \frac{\omega_l^2 \omega_{pe}^2}{k^2 (\omega_l^2 - k_z^2 v_{te}^2)} \left[\frac{k_y}{L \omega_l |\omega_{ce}|} + \frac{k_\perp^2}{\omega_{ce}^2} - \frac{k_z^2}{\omega_l^2} \right], \quad (2.9)$$

and

$$\chi_i = \frac{1}{k^2 \lambda_{Di}^2} \left[1 - \sum_{n=-\infty}^{\infty} \frac{\omega_l \Delta}{\omega_l - n \omega_{ci}} \left[1 - \frac{\omega_i^*}{\omega_l} \left(1 - \frac{n \omega_l}{b_i \omega_{ci}} \right) \right] \right], \quad (2.10)$$

where $\lambda_{Di} = v_{ti} / \omega_{pi}$ is the ion Debye length, v_{ti} is the ion thermal velocity, ω_{pi} is the ion plasma frequency, $\Delta = I_n(b_i) \exp(-b_i)$, $b_i = k_\perp^2 \rho_i^2$, $\rho_i = v_{ti} / \omega_{ci}$, $\omega_i^* = -k_y v_{ti}^2 / \omega_{ci} L$, and I_n is the modified Bessel function of order n .

Eliminating ϕ_l from (2.3) and (2.8), we obtain

$$D_t \epsilon_l = -q_e^2 \omega_{pe}^4 k_\perp^2 | \phi_0 |^2 / 2 m_e^2 \omega_l^4 (\omega_0 - \omega_{ce})^2 k^2, \quad (2.11)$$

which is our new nonlinear dispersion relation. In the absence of the UH pump, the LHCP electromagnetic wave and the low-frequency electrostatic modes appear as independent eigenmodes of the inhomogeneous magnetized plasma. The corresponding linear dispersion relations are $D_t = 0$ and $\epsilon_l = 0$. The latter gives the low-frequency mode. For example, for $\omega_l \gg k_z v_{te}$, ω_{ci} , $b_i \ll 1$, and $\omega_{ci} \ll \omega_l k_y L$, we have [Yu et al., 1985] $\omega_l = -(\omega_{pi}^2/2\omega_v \Delta) \pm [(\omega_{pi}^4/4\Delta^2 \omega_v^2) + \omega_{LH}^2]^{1/2}$, where $\omega_v = k^2 L \omega_{ci}/k_y$, $\omega_{LH}^2 = (\omega_{pi}^2/\Delta)(1 + m_i k_z^2/m_e k_\perp^2)$, $\Delta = 1 + \omega_{pe}^2/\omega_{ce}^2$, and $k_\perp \gg k_z$ has been assumed. Clearly, the standard lower-hybrid waves $\omega_l = \pm \omega_{LH}$ exist only if $k_y L \Delta^{1/2} \gg \omega_{pi} k_y^2/\omega_{ci} k_\perp^2$. When the inequality is reversed, such as, in a region of strong density gradient, the forward propagating lower-hybrid waves (positive sign) turn into the electron convection waves whose frequency is given by $\omega_l = \omega_v(1 + m_i k_z^2/m_e k_\perp^2)$.

On the other hand, in the limits $\omega_l \ll k_z v_{te}$, ω_{ci} , $k_\perp^2 - k_y \omega_{ce}/L \omega_l \ll (\omega_{ce} k_z/\omega_l)^2$, $b_i \ll 1$, we have $\chi_e \approx 1/k^2 \lambda_{De}^2$ and $\chi_i \approx (\omega_{pi}^2 k_\perp^2/\omega_{ci}^2 k^2)(1 - \omega_{ci} k_y/\omega_l k_\perp^2 L)$, where $\lambda_{De} = (\epsilon_0 k_B T_e/n_0 q_e^2)^{1/2}$ is the electron Debye length (k_B is the Boltzmann constant). For this case, the low-frequency long wavelength (in comparison with λ_{De}) longitudinal mode would be the usual electrostatic drift wave, given by $\omega_l \approx k_y v_{te}^2/\omega_{ce} L(1 + k_\perp^2 \rho_s^2)$, where $\rho_s = (T_e/m_i \omega_{ci}^2)^{1/2}$ is the ion Larmor radius at the electron temperature.

Assuming that $D_t \approx 0$ and $\epsilon_l \approx 0$, we can Taylor expand the left-hand side of (2.11) and obtain the growth rate

$$\gamma = \frac{q_i}{\sqrt{2} m_e} \frac{\omega_{pe}^2}{\omega_l^2} \left| \frac{C}{\omega_0 - \omega_{ce}} \right| \left| \left(\frac{\partial D_t}{\partial \omega_t} \right) \left(\frac{\partial \epsilon_l}{\partial \omega_l} \right) \right|^{-1/2} |\phi_0|. \quad (2.12)$$

The threshold value is obtained by incorporating the proper linear damping rates of the radiation and the lower-hybrid-convection mode. We find

$$|\phi_0|^2 = 2 \frac{m_e^2}{q_i^2} \frac{\omega_l^4}{\omega_{pe}^4} (\omega_0 - \omega_{ce})^2 \frac{\partial D_t}{\partial \omega_t} \frac{\partial \epsilon_l}{\partial \omega_l} \frac{\gamma_t \gamma_l}{C^2}, \quad (2.13)$$

where γ_t and γ_l are, respectively, the damping rates of the transverse and low-frequency longitudinal waves [e.g. Murtaza and Shukla, 1984]. Thus, the rate of generation of radiation by the UH pump in nonuniform magnetoplasmas is governed by (2.12) provided that the threshold electric field satisfies (2.13). We observe that the effect of the plasma nonuniformity is to modify the strength of the coupling constant, as well as the spectrum of the low-frequency longitudinal waves.

3 Summary

In summary, we have presented a nonlinear theory for the generation of radiation in a planetary magnetosphere. Specifically, we have considered the parametric decay of a monochromatic UH electrostatic pump into a LHCP electromagnetic wave and a low-frequency longitudinal wave in an inhomogeneous magnetized plasma. For this purpose, a new dispersion relation for the resonant three-wave decay interaction has been derived and analyzed. We found that the plasma inhomogeneity introduces new low-frequency

electrostatic modes, which may now participate in the decay interaction process. The present nonlinear process may account for the continuum radiation that has been observed in the magnetospheres of the Earth as well as other planets.

Finally, we mention that the present investigation can be easily generalized for cases in which the O- and X-modes are involved. The electromagnetic wave spectra as well as the strength of the coupling coefficients would then be different from those presented here.

Acknowledgments: The authors thank K. Rönmark for stimulating discussions. This research was performed under the auspices of the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 191 "Physikalische Grundlagen der Niedertemperaturplasmen" and the Deutsche Forschungsanstalt für Luft- und Raumfahrt(DLR)/Indian Space Research Organization (ISRO) Exchange Program.

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