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Uncertainties in phase shifts and amplitude ratios: Theory and practice

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Abstract

Multicolor data of pulsating variables yield information on the amplitude ratios and phase shifts between the different passbands. This is a powerful tool for mode identifications of radial and nonradial pulsators. The identifications rely on relatively small effects, so that the uncertainties due to measurement errors need to be known precisely.

We present the formulae which allow the calculation of the statistical uncertainties from the residuals between the measurements and the least-squares fit of sinusoids to the light curves. Since it has been often presumed that the real uncertainties of the amplitudes and phases are greater than calculated from statistics, we have compared the theoretical scatter with the observed scatter. Nine pulsation modes of the δ Scuti variable 44 Tau, extensively observed for five observing seasons, were chosen. The observed and predicted scatter of 86 pairs of amplitude ratios and phase shifts were compared.

We find that the observed and predicted scatter are very similar: the histograms of the observed scatter in the amplitude ratios and phase shifts match the normal distribution predicted from the formulae. The excellent agreement might be a consequence of the fact that most systematic observational and computational (caused by multiperiodicity) errors tend to cancel out when the measurements at the different passbands are compared.

Individual Objects: 44 Tau

Introduction

The identification of the excited pulsation modes forms the basis of observational asteroseismology. The light curves of the multiperiodic pulsators are generally used to detect the multiple frequencies of pulsation together with their amplitudes and phases. If multicolor light curves are available, these also form powerful tools to identify the pulsation modes (particularly the ℓ values). For the mode identification, two parameters for each frequency are especially important: the amplitude ratios and phase shifts between two carefully chosen passbands. The identifications rely on relatively small effects, so that the uncertainties due to measurement errors need to be known precisely. In fact, only the recent developments of large observational campaigns with high precision have allowed the reliable determination of the small phase shifts.

The solutions to the observed light curves involve multiple sinusoidal fits, which are usually obtained by least-squares algorithms. A number of different statistical packages are used by different research groups. An example is the package PERIOD04 (Lenz & Breger 2005), which computes amplitudes and phases together with the formal uncertainties of these fits. The question arises whether the computed values of the uncertainties are realistic. Montgomery & O'Donoghue (1999) wrote, "the naive least-squares formulae provide only a lower limit to the errors, and the true errors may be much higher." It has been the author's experience that the photometric data obtained for the same stars in different years often show uncertainties up to 50% higher than predicted by the simple formulae (given in the next section). This is due to the fact that observational noise is correlated. In the case of δ Scuti stars, another effect comes into play: the star itself changes the frequency values (i.e., the phases) and amplitudes, often by small amounts. For many years we have been following the star 4 CVn for more than 100 nights/year in order to examine the nature of this variability. The currently unpublished results indicate intrinsic 'jitter' even in the modes with high amplitudes.

When we turn to multicolor data and consider the derived values of amplitude ratios and phase shifts between different passbands, the situation changes. Most two-color measurements are obtained with the same instrument and almost simultaneously. This means that several components of the observational noise cancel out (or their effects are reduced) since they affect both passbands in a similar manner. Of course, photon statistics is not cancelled; this noise source is not correlated from measurement to measurement. Also, the phase and amplitude drifts intrinsic to the star as well as any effect of unrecognized additional frequencies should cancel out. Consequently, the bad situation described in the previous paragraph may not apply to the amplitude ratios and phase differences determined under near-identical conditions for the two colors.

In this paper, we want to compare the predicted and observed scatter in the amplitude ratios and phase shifts for the star 44 Tau: this is one of the few stars (if not the only one) for which five extremely extensive sets of data from different years are available (Breger et al. 2008) and for which the comparison can be carried out.

Theoretical statistical uncertainties

The light curves of many types of pulsating stars (such as δ Scuti stars) are nearly sinusoidal so that they can be mathematically described by a sinusoid with frequency ω_1. The slight asymmetries, in practice, are taken care of by including a $2\omega_1$ term for the modes with the largest amplitudes.

Suppose we have N measurements of the magnitudes, m_i, at times t_i. We assume that the times of the observations are error free, but that the brightness measurements are subject to random errors, Δm_i, which have an average of zero, a constant root-mean-square amplitude, and are not correlated in time.

In order to analyze our time series data, we fit a sinusoid to it. Specifically, we fit the function

$$f(t) = a_0 + a\sin(\omega t_i + \phi), \tag{1}$$

where the frequency, ω, is assumed to be known, but where the amplitude a and phase ϕ need to be determined. This is a realistic situation when we wish to compare the amplitudes and phases for data measured in two passbands. The parameter a_0 represents a constant offset.

We define

$$\begin{aligned} \chi^2 &\equiv \sum_{i=1}^{N} [m_i - f(t_i)]^2 \\ &= \sum_{i=1}^{N} [m_i - a_0 - a\sin(\omega t_i + \phi)]^2 , \end{aligned} \tag{2}$$

where the minimum in χ^2 corresponds to the best fit solution of the model parameters. We now minimize χ^2 with respect to a_0, a, and ϕ and derive the following relations (for details see Breger et al. 1999, Montgomery & O'Donoghue 1999):

$$\sigma(a) = \sqrt{\frac{2}{N}} \cdot \sigma(m), \tag{3}$$

which is the desired relation between photometric and amplitude uncertainties. Here $\sigma(m)$ is the standard deviation of the measured brightnesses relative to the fits.

We also find

$$\sigma(\phi) = \frac{\sigma(a)}{a} = \sqrt{\frac{2}{N}}\frac{\sigma(m)}{a}, \tag{4}$$

which is the desired relation between the photometric error, the amplitude of the signal, and the error in the phase determination.

It is common among observers to express ϕ in degrees and to relate the uncertainties in amplitude and phase. The equation then becomes

$$\sigma(\phi) = 57.3\ \sigma(a)/a \tag{5}$$

Suppose that the data have been obtained through two separate filters, 1 and 2. Let us assume that the measurements in the two passbands are independent of each other and the errors are not correlated.

When we carry out the error propagation, we find

$$\sigma(\frac{a_1}{a_2}) = \frac{a_1}{a_2}\sqrt{(\frac{\sigma(a_1)}{a_1})^2 + (\frac{\sigma(a_2)}{a_2})^2} \tag{6}$$

$$\sigma(\phi_1 - \phi_2) = \sqrt{(\frac{\sigma(a_1)}{a_1})^2 + (\frac{\sigma(a_2)}{a_2})^2}, \tag{7}$$

or in degrees,

$$\sigma(\phi_1 - \phi_2) = 57.3\sqrt{(\frac{\sigma(a_1)}{a_1})^2 + (\frac{\sigma(a_2)}{a_2})^2}. \tag{8}$$

This allows us to calculate the uncertainties in the amplitude ratio and phase shift.

Many pulsating stars are multiperiodic so that Eqn. 1 needs to be replaced by

$$f(t) = a_0 + \sum_{j=1}^{F} a_j \sin(\omega_j t_i + \phi_j), \tag{9}$$

where F represents the number of frequencies (and harmonics).

We can only apply the equations derived earlier to each of the frequencies if the data are sufficient to ensure that the different frequencies do not influence each other's solution. This is not fulfilled for small data sets. However, such small data sets would not be used anyhow to give astrophysically meaningful values of phase shifts and amplitude ratios. Montgomery & O'Donoghue (1999) applied a test to six-frequency solutions of the 1996 data of 4 CVn and found a deviation of the errors of no more than 12%. Since most of the recent available campaigns are much more extensive than the data set used by Montgomery & O'Donoghue, we do not consider the multiple frequencies as a severe problem.

Table 1: Example of a comparison of phase shifts (radial mode at 8.96 c/d)

Observing season	Phase shift	Difference, d, between years	
	degrees	degrees	normalized
2004/5	2.93 ± 1.02		
2005/6	2.46 ± 0.93	0.47 ± 1.38	0.34

Observed uncertainties in in phase shifts and amplitude ratios

Recently, a very large data set has become available, which allows us to examine the validity of the uncertainties computed from the residuals between the measurements and the fits, i.e., δm. The star 44 Tau was studied extensively for five years from 2000/1 to 2005/6 (Breger & Lenz 2008) in two colors, viz., the Stromgren v and y passbands. We selected nine frequencies with relatively large amplitudes. A tenth frequency at 9.56 c/d was not used because of the presence of a close frequency companion at 9.58 c/d and the possible contamination. Some of the frequencies show strong amplitude variability from year to year. This caused us to reject one of the forty-five available solutions: in 2005/6, the mode at 9.12 c/d had a near-zero amplitude.

For each observing season and frequency, we computed the amplitude ratio, v/y, and phase shift, $\phi(v) - \phi(y)$. We also calculated the formal uncertainties of these values, which differ because of the different number of observations, annual residuals of the fits and variable amplitudes of pulsation. We then compared the results for each year with those of each of the other years. This resulted in ten comparisons for each frequency. The differences in the measured values, d, were then normalized to the 'expected' standard deviations computed from the known residuals of the annual solutions, $\sigma(m)$. We obtained 86 ratios for each of the amplitude ratios and phase shifts. These should ideally follow a normal distribution. Our approach is illustrated in Table 1, which shows one of the 86 calculations.

We then examined the distribution of the 86 amplitude-ratio and phase-shift differences. Since each value was already scaled (normalized) to the predicted standard deviation, a perfect fit of the statistical formulae discussed in the previous section would lead to a normal distribution of these 86 values. The actual distributions are shown in Fig. 1 and 2. The bin size was chosen to give a sufficiently high number of occurrences, B, in order for the uncertainties ($=\sqrt{B}$) not to dominate. We found that, visually, the agreement with the predicted normal distribution (shown as curves) is excellent. If we fit Gaussian curves to the data and examine the standard deviations corresponding to these

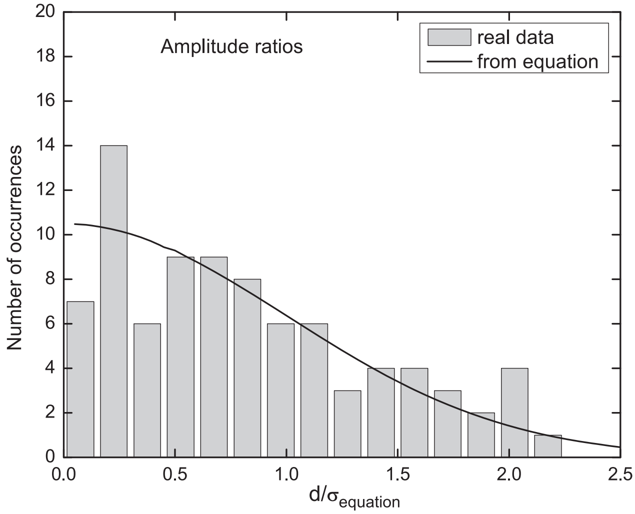

Figure 1: Histogram of the observed differences in the calculated amplitude ratios between different years for nine frequencies of 44 Tau. The differences were normalized relative to the standard deviations expected from the theoretical uncertainties. The drawn curve represents the expected Gaussian distribution. Each observed number of occurrences has a formal uncertainty of the square root of its value.

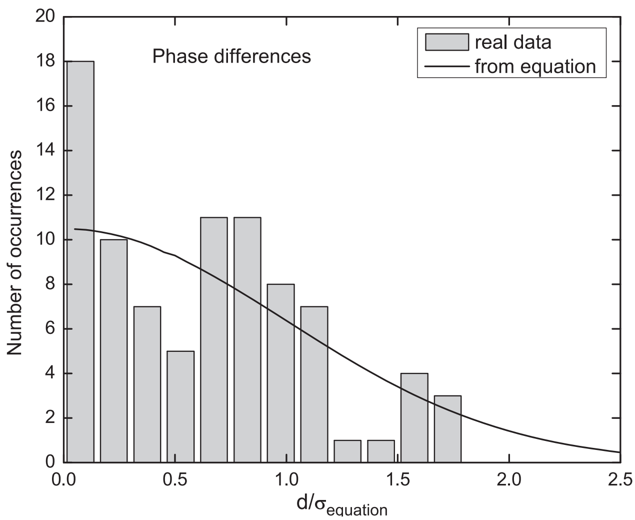

Figure 2: Same as Fig. 1 for the observed phase shift, $\phi(v) - \phi(y)$. This diagram shows that the observed and theoreticallypredicted scatter are similar.

curves, the amplitude ratios have slightly larger standard deviations, while the phase shifts show slightly smaller values. In both cases, the deviations from 1.0 are not statistically significant.

Finally, it needs to be noted that the present examination dealt with the statistics of phase shifts and amplitude ratios, when the variations in different passbands are compared. Since these two-color measurements are collected almost simultaneously with the same instrument, many systematic observational errors and computational errors (caused by multiperiodicity) affect both sets of measurements similarly. They, therefore, tend to cancel out. This is not the case for the light curves in a single color: the absolute values of the amplitudes and phases of the fit may still be less accurate than predicted by the formulae.

We conclude that for the data sets tested, the theoretical uncertainties of the amplitude ratios and phase shifts correspond very well to the observed values. When the measurements in the two colors are obtained under similar observing conditions, there is no need to artificially increase the values of the computed statistical uncertainties.

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