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The Source of Nicholas Rhabdas' *Letter to Khatzykes*: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4

Abstract: The article presents the edition and a detailed discussion of an anonymous treatise of elementary arithmetic that served as the source of the so-called *Letter to Khatzykes* authored by the Byzantine scholar Nicholas Artabasdos Rhabdas. An updated survey of the extant evidence about the logistic treatises composed in the Nicaean period and in the early Palaiologan era, and a discussion of the *prima facie* surprisingly widespread phenomenon of appropriation of scientific treatises written by other contemporaries in late Byzantine times will also be provided.

This article presents the edition of an anonymous treatise of elementary arithmetic (henceforth called "*Anonymus* B") contained, in slightly incomplete form, in ff. 171r–186v of the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, Barberinianus gr. 4 (*Diktyon* 64552), to be dated to the beginning of the 14th century. The most important point of our study, however, does not lie in assessing *Anonymus* B in its own terms, but in showing that it served as the source of the so-called *Letter to Khatzykes*[1] authored by the Byzantine scholar Nicholas Artabasdos Rhabdas[2]. That the *First Letter* and the anonymous treatise are very closely connected is obvious from their verbatim agreeing over large stretches of text; two series of facts decisively support our stronger claim about their filiation. First, the Barberini codex, produced within the circle of Maximos Planudes' (d. ca. 1305) pupils, dates at the latest to the period of Rhabdas' early activity. Second—and crucially, on account of the fact that the paleographical record does not completely settle the issue of priority—the involved variant readings, starting from the very titles of the two texts, strongly corroborate the hypothesis that *Anonymus* B is the source of Rhabdas' *First Letter*, and not the inverse. As it also happens that an anonymous arithmetical treatise dated 1252 (henceforth "*Anonymus* 1252") underwent a similar treatment in the hands of Planudes himself, resulting in his celebrated *Great Calculation According to the Indians*, we shall provide a revised outline of the extant evidence about the logistic treatises redacted in the Nicaean period (1204–61) and in the early Palaiologan era; we shall also argue, on grounds of style and contents, that *Anonymus* B and *Anonymus* 1252 were not composed by the same author. Consequently, we shall discuss the *prima facie* surprisingly widespread phenomenon, in late Byzantine times, of appropriation of scientific treatises written by other Byzantine scholars.

The structure of the article is as follows. We first survey the evidence about Rhabdas' scientific career; we then briefly describe Barb. gr. 4. The edition of *Anonymus* B, including the accompanying tables, is followed by an analysis of the textual differences with respect to Rhabdas' *First Letter*, and by an assessment of the flourishing of logistic treatises in the middle of the 13th century. In the last

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[1] Henceforth called *First Letter*. We shall see that there are two "letters" of arithmetical content authored by Rhabdas.

[2] Rhabdas was born in Smyrna and was active in Constantinople about 1320–40; see *PLP*, no. 1437; *ODB*, 1786–1787; and the discussion in the next section.

section, we present factual evidence and some considerations on the issue of "scientific plagiarism" in Palaiologan Byzantium.

NICHOLAS RHABDAS, LIFE AND WORKS

Enough of Nicholas Rhabdas' scholarly production has been preserved for us to acknowledge his expertise in the mathematical sciences and especially his significant contribution to the domain of Byzantine logistic[3]. This is a branch of arithmetic in which a unit can be divided and that deals with counting numbers and with calculations on them[4]. Logistic developed greatly in Late Antiquity as a support to mathematical astronomy, and retained this role in Byzantine times[5]. As we shall see, Rhabdas was also engaged in astronomical matters and, in addition, composed a grammatical treatise for his son Paul Artabasdos[6].

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[3] His production, however, was assessed in a way that is paradigmatic of a generalized dismissive attitude to Byzantine science; P. Tannery, Manuel Moschopoulos et Nicolas Rhabdas. *Bulletin des Sciences mathématiques*, 2e série, 8 (1884) 263–277, repr. Id., Mémoires Scientifiques IV. Toulouse–Paris 1920, 1–19: 15, in fact, passed the following judgment on Rhabdas' writings, whose edition he nevertheless published two years later: "L'intérêt de ses écrits est surtout de montrer jusqu'où étaient tombés les héritiers dégénérés du nom hellène, ceux-là même qui avaient alors Diophante entre leurs mains".

[4] According to the 6th-century Neoplatonic commentator Eutocius, dividing the unit does not pertain to arithmetic but to logistic (J. L. Heiberg (ed.), Archimedis opera omnia cum commentariis Eutocii. I–III. Lipsiae 1910–15, III 120.28–30: ὥστ' ἐπ' ἐκείνων [*scil.* superparticular and superpartient ratios] διαιρετέον τὴν μονάδα, ὃ εἰ καὶ μὴ κατὰ τὸ προσῆκον τῇ ἀριθμητικῇ ἀλλὰ τῇ λογιστικῇ τυγχάνει "so that, for them one has to divide the unit, even if this does not happen to fit to arithmetic, but to logistic"). An earlier definition of logistic—which can almost certainly be ascribed to Geminus (a 1st-century BCE mathematically-minded philosopher, maybe a pupil of Posidonius)—does not allow dividing the unit. This definition can be found in pseudo-Hero, *Def.* 135.5–6 (J. L. Heiberg – L. Nix – W. Schmidt – H. Schöne (eds.), Heronis Alexandrini opera quae supersunt omnia. I–V. Lipsiae 1899–1914, IV 98.12–100.3), and is also preserved, through a different line of tradition, as a scholium to Plato, *Chrm.* 165e6 (Scholium 27 in D. Cufalo, Scholia Graeca in Platonem. I. Scholia ad dialogos tetralogiarum I–VII continens [*Pleiadi* 5.1]. Roma 2007, 173). It is possible that the domain of logistic was expanded to include fractional parts as a consequence of the adoption of the sexagesimal system in Greek mathematical astronomy. In fact, logistic developed greatly in Late Antiquity as a support to mathematical astronomy, and also played this same role in the Byzantine period. The first known treatise of this kind is included in the *Prolegomena to the Almagest*, and amounts to the (non-redacted) lecture notes of a course held in the circle of the Neoplatonic philosopher Ammonios. This treatise is a computational primer to the *Almagest*: a tightly organized "handbook of logistic" featuring as its main themes an introduction to the sexagesimal system, a description of computational algorithms for multiplication, division, and extraction of an approximate square root, a presentation of interpolation techniques, and an exposition about compounded ratios and removal of a ratio from a ratio. According to the anonymous author, no comprehensive previous exposition of this kind existed—and in fact no such one has been transmitted to us. See also note 125 below. The best introduction to Greek logistic is still K. Vogel, Beiträge zur griechischen Logistik. Erster Teil (*Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Abteilung*). Munich 1936, 357–472.

[5] Cf. the explicit statement opening *Anonymus* 1252: A. Allard, Le premier traité byzantin de calcul indien: classement des manuscrits et édition critique du texte. *Revue d'Histoire des Textes* 7 (1977) 57–107: 80.2–4, and, in a smoother formulation, Planudes' *Great Calculation*: A. Allard (ed.), Maxime Planude, Le grand calcul selon les Indiens. Louvain-la-Neuve 1981, 27.1–5. Despite its title (and the author's statement similar to that of Planudes: P. Carelos [ed.], Βαρλαὰμ τοῦ Καλαβροῦ, Λογιστική, Barlaam von Seminara, Logistiké [*Corpus philosophorum Medii Ævi. Philosophi byzantini* 8]. Athens–Paris–Brussels 1996: 1.10–26), Barlaam' *Logistic* is not a writing of logistic, but a fully-fledged treatise of theoretical arithmetic formulated in an impeccable demonstrative style. Barlaam (*PLP*, no. 2284), undisputably the Byzantine scholar best versed in mathematical matters and a major actor in the hesychastic controversy, died in 1348.

[6] See *PLP*, no. 1438. The unpublished grammatical synopsis addressed to Paul is preserved in the miscellaneous ms. Paris, Bibliothèque nationale de France, gr. 2650 (*Diktyon* 52285), ff. 147r–150v. The copying is dated December 6, a.m. 6836 [= 1427] (f. 204v), certainly referring to ff. 201r–204v and possibly also to ff. 153r–167v and 168v–198v, penned by the same hand. However, the script of Rhabdas' synopsis (located in quire no. ιθ´, a ternion closed by the blank ff. 151r–152r) seems earlier, perhaps dating back to the middle – third quarter of the 14th century. The synopsis is presented as a grammatical compendium whose aim is expounding the appropriate use of words, in order to avoid barbarisms and solecisms. The exposition is based on analytical divisions of the main grammatical issues, treated by means of μικρούς τινας ὑπομνηματισμούς "some short notes". It starts from letters (γράμματα and στοιχεῖα) and goes on dealing with syllables and words insofar as they are

Much less is known about Rhabdas' life, education, personal and professional networks. Until recently, the only temporal clue was provided by the fact that, in the Easter Computus expounded in his *Letter to Tzavoukhes* (see below), 1341 is assumed as the current year. More recently, however, A. Riehle has proposed a new periodization of Rhabdas' lifetime pushing the date of his birth as early as ca. 1295[7]. Riehle's dating is based on the evidence provided by a previously unedited letter addressed to Andronikos Zarides (ca. 1275–after 1327; *PLP*, no. 6461), in which Rhabdas informs his addressee that a partial solar eclipse will occur on June 26, 1321, while a lunar eclipse will take place on July 10, 1321[8]. Rhabdas provides the hours in which the Sun and the Moon will be eclipsed and expresses hopes that his calculations are correct[9]. Using the dating of the letter to Zarides (1321), its style and content (suggesting that Rhabdas was still a young man when he composed it), the level of astronomical expertise demonstrated in the letter, and the dedication of Manuel Moschopoulos'[10] treatise on magic squares to Rhabdas[11], Riehle has concluded that the latter cannot be born later than 1295[12]. Riehle also suggested the possibility of identifying Rhabdas with the promising student from Smyrna mentioned in an anonymous and unedited letter to an equally unnamed addressee preserved in Laur. Plut. 59.35, f. 183r, namely, in the same manuscript transmitting Rhabdas' letter to Zarides. In addition to his native town of Smyrna, Rhabdas must have resided at least temporarily in Constantinople, since his correspondence and the titles of his logistic treatises both indicate Constantinople as Rhabdas' residence. The identification of Rhabdas' teachers can only be a matter of speculation.

Through Andronikos Zarides and Manuel Moschopoulos, both pupils of Maximos Planudes, Rhabdas might have been connected to the latter's circle and school. The Planudean connection carries through Rhabdas' scientific output as he prepared a slight revision—including a couple of

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the usual μέρη τοῦ λόγου. Title: γραμματικῆς σύνοψις ἠκριβωμένη | ζητεῖ μαθεῖν πόνημα τίς τίνος τόδε, ἐξ Ἀρταβάσδων τοῦ Ῥαβδᾶ Νικολάου, *inc*. τῆς γραμματικῆς τέχνης πολυσχιδοῦς καὶ ἀπείρου σχεδὸν ἐούσης (*sic*), φίλτατε υἱὲ Ἀρτάβασδε Παῦλε, *des*. καὶ διὰ κανόνος ἑνὸς τοῦ βωκὸς τὴν κλίσιν τούτων ὁμοίαν γνωρίσας ἐν τούτῳ τελειοῖ τὴν εἰς ξ κατάληξιν.

7 A. Riehle, Epistolographie und Astronomie in der frühen Palaiologenzeit. *JÖB* 65 (2015) 243–252, on which we partly rely in this paragraph.

8 Discussion of the astronomical data in A. Tihon, Nicolas Eudaimonoioannes, réviseur de l'Almageste? *Byzantion* 73 (2003) 151–161: 153–154; Riehle, Epistolographie 246; edition of the letter ibid., 251. The letter can only be found in the ms. Firenze, Biblioteca Medicea Laurenziana, Pluteus 59.35 (*Diktyon* 16486) (ca. 1325), f. 204r–v. In 1321, the approximate dating of the letter, Zarides was in Thessalonike; on this and on Zarides, see A. Cohen-Skalli – I. Pérez Martín, La Géographie de Strabon entre Constantinople et Thessalonique: à propos du Marc. gr. XI.6. *Scriptorium* 71 (2017) 175–207: 195–197.

9 Thus, Rhabdas' letter to Zarides compares with the letters his contemporary Nikephoros Gregoras (d. ca. 1360) addressed to John Chrysoloras and Michael Kaloeidas in order to discuss the calculation of both past and future solar eclipses: *Epistulae*, no. 53, 103 ed. P. A. M. Leone, Nicephori Gregorae epistulae. I–II. Matino 1982 (= no. 33, 51 ed. R. Guilland, Correspondance de Nicéphore Grégoras. Paris 1927). Gregoras also redacted a tract on the solar eclipse of July 16, 1330 (ed. J. Mogenet – A. Tihon – R. Royez – A. Berg, Nicéphore Grégoras, Calcul de l'éclipse de soleil du 16 juillet 1330 [*Corpus des Astronomes Byzantins* 1]. Amsterdam 1983) transmitted with autograph corrections in the ms. Venezia, Biblioteca Nazionale Marciana, gr. Z. 325 (*Diktyon* 69796).

10 On Moschopoulos, who died after ca. 1306, see *PLP*, no. 19373, and C. Constantinides, Higher Education in Byzantium in the Thirteenth and Early Fourteenth Centuries (1204 – ca. 1310) (*Texts and Studies of the History of Cyprus* 11). Nicosia 1982, 103–108. As a matter of fact, the title of Moschopoulos' treatise gives more prominence to Rhabdas than to its author: the former is qualified ἀριθμητικὸς καὶ γεωμέτρης "arithmetician and geometer", whereas Moschopoulos is simply λογιώτατος καὶ μακαριώτατος "most learned and most happy"—and thus he was already dead—and he redacted the treatise βιασθείς "spurred on" by Rhabdas. All of this strongly suggests that Rhabdas himself took care of the edition after Moschopoulos' death. As we shall see, it is highly significant in this respect that the most authoritative manuscript witness of Moschopoulos' treatise is Vat. gr. 1411 (*Diktyon* 68042).

11 See P. Tannery, Le traité de Manuel Moschopoulos sur les carrés magiques. Texte grec et traduction. *Annuaire de l'Association pour l'encouragement des études grecques en France* (1886) 88–118, repr. Id., Mémoires scientifiques IV. Toulouse–Paris 1920, 27–60: 32.1–4. An analysis of Moschopoulos' treatise is in J. Sesiano, Les carrés magiques de Manuel Moschopoulos. *Archive for History of Exact Sciences* 53 (1998) 377–397; see also Tannery, Moschopoulos et Rhabdas.

12 Riehle, Epistolographie 246–248. For arguments against setting Rhabdas' birthdate earlier than 1295, see ibid., 247 n. 31.

short additions—of Planudes' *Great Calculation*. This recension is transmitted in a number of manuscripts; other, disparate texts were added to it during the 14th century[13].

Direct evidence of Rhabdas' connection with Nikephoros Gregoras comes from a square roots table[14], preserved in the ms. Heidelberg, Universitätsbibliothek, Palatinus gr. 129 (*Diktyon* 32460) (mainly 14th century), ff. 11v–12r, comprising an additional bifolium[15]. The current inscription of Rhabdas' table only acknowledges its content and its author as it pens with black ink ὅρα πλευρὰς ἀρρήτους τοῦ Ῥαβδᾶ Νικολάου "here, unexpressible square roots by Nicholas Rhabdas". This inscription, however, is the result of an intrusive revision of the original title, written in red ink by the same scribe who copied the table, which identified Gregoras as the dedicatee and was transcribed by Biedl as follows: πλευρὰς ἀρ{ρ}ήτους, Γρηγορᾶ σοφέ, δέχου ἐξ Ἀρταβάσδου τοῦ Ῥαβδᾶ Νικολάου "accept, wise Gregoras, unexpressible square roots from Nicholas Artabasdos Rhabdas". The clause was subsequently partly covered with black ink and revised as indicated above; the outcome is to omit mentioning the dedicatee Gregoras, thus erasing in an act of *damnatio memoriae* the only place the latter's name figured in a codex he himself compiled and owned. The square roots table ranges from 1 to 120 but, for reasons that escape us, its second half is empty; thus it is unlikely that the bifolium Gregoras added was the actual table sent to him by Rhabdas. The table, the values in which are given in the sexagesimal system and are approximated to second minutes, is not calculated according to the procedure expounded in Rhabdas' *Letter to Tzavoukhes*.

Rhabdas' scientific production includes two logistic treatises in the form of "letters", namely, the so-called *Letter to Khatzykes* and *Letter to Tzavoukhes*. While nothing else is known of Theodore Tzavoukhes of Klazomenai (*PLP*, no. 27609), a richer surviving evidence concerns George Khatzykes (*PLP*, no. 30724). He served under Andronikos II as προκαθήμενος τοῦ κοιτῶνος (1305–

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[13] That Rhabdas authored a revision of Planudes' treatise is borne out by its title in the manuscripts themselves. Again, the title appears to downplay Planudes' contribution, clearly alluding to the fact that a source is to be understood (Vat. gr. 1411, f. 122r): Ψηφηφορία κατ' Ἰνδοὺς ἡ λεγομένη μεγάλη. ταύτης ἡ φράσις τοῦ φιλοσοφωτάτου ἐν φιλοσόφοις καὶ τιμιωτάτου ἐν μοναχοῖς κυροῦ Μαξίμου τοῦ Πλανούδη καὶ τοῦ Ῥαβδᾶ Νικολάου *Great Calculation According to the Indians. This formulation of it is by the most scholarly scholar and most honourable monk Maximos Planudes and by Nicholas Rhabdas*. The main witness of Rhabdas' revision of Planudes' treatise is in fact Vat. gr. 1411 (early 15th century, copyist <John Eugenikos>: A. Gioffreda), in which the revision is possibly (but not necessarily) incomplete, as it ends in the middle of f. 126v, at Allard, Planude 61.8 εἴρηται. The Vatican manuscript is important since P. Tannery (Notice sur les deux lettres arithmétiques de Nicolas Rhabdas. *Notices et extraits des manuscrits de la Bibliothèque Nationale* 32.1 [1886] 121–252, repr. Id., Mémoires scientifiques IV. Toulouse–Paris 1920, 61–198: 73, 76, and 82–83) surmised, rightly in our opinion, that it was a copy of a codex, maybe of logistic and arithmetical contents, conceived and realized by Isaak Argyros (*PLP*, no. 1285) and, before him and in part, by Rhabdas. Tannery's contention is based on the title of Moschopoulos' treatise on magic squares, on the presence, in Vat. gr. 1411, of three short arithmetical texts ascribed to Argyros, of *recensio* II of Philoponos' commentary on Nikomachos' *Introduction*—a recension Tannery ascribes to Argyros himself—and of two additions to Rhabdas' additions (!) to his revision of Planudes' *Calculation*, marked in the margin by a mysterious τοῦτο ἡμέτερον (Vat. gr. 1411, ff. 123v and 125v). The edition of Rhabdas' additions is in Allard, Planude 203–211. On this issue and on Vat. gr. 1411, see now F. Acerbi, I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro. *Estudios bizantinos* 5 (2017) 131–206.

[14] See A. Biedl, Der Heidelberger cod. Pal. gr. 129 — die Notizensammlung eines byzantinischen Gelehrten. *Würzburger Jahrbücher für die Altertumswissenschaft* 3 (1948) 100–106: 104–106 (the inscriptions of the table are also transcribed), and I. Pérez Martín, El Escurialensis X.I.13: una fuente de los extractos elaborados por Nicéforo Gregorás en el Palat. Heidelberg. gr. 129. *BZ* 86–87 (1994) 20–30. See also D. Bianconi, La biblioteca di Cora tra Massimo Planude e Niceforo Gregora. Una questione di mani. *Segno e Testo* 3 (2005) 391–438: 412. Relying on the original inscription of Rhabdas' table and on its presence in a volume compiled and partially copied by Gregoras himself (but the original inscription is not in Gregoras' hand), Biedl concluded that Rhabdas and Gregoras must have been close associates. The Heidelberg manuscript can be accessed at http://digi.ub.uni-heidelberg.de/diglit/cpgraec129.

[15] The bifolium consisting of ff. 11 and 12 appears to be an addition that was first appended to the body of the codex and then completed with disparate texts. On f. 11r, Gregoras copied several excerpts from Plutarch, whereas f. 12v features a rudimentary addition table. On f. 12v we can see the reinforcement flap added along the spine, with an annotation in Gregoras' hand; this means that he was probably the one who secured the incorporation of the bifolium. However, he did not copy Rhabdas' table.

10) and as ἐπὶ τῶν δεήσεων (until 1325); he corresponded with Manuel Gabalas and, like Zarides, with Michael Gabras.

The *Letter to Khatzykes* contains the following (references are to the pages of Tannery's edition)[16]: denominations of numbers and how to represent integers from 1 to 9,999 on the fingers of the hands (86.1–96.12)[17]; abstract descriptions of the five elementary arithmetic operations on integers, extraction of an approximate square root included (96.13–102.9); denominations of numerical orders and their multiplication (102.10–110.5)[18]. A structured set of tables of addition, subtraction, multiplication, and partition is found at the end of the treatise and was apparently meant to complete it; it also contains an introduction to the partition table (114.1–17)[19]. This is the (short) treatise whose source is *Anonymus* B. One must stress that no arithmetic operation is actually carried out, and that no instructions for use are provided for the tables.

Next, the *Letter to Tzavoukhes*[20] contains the following: multiplication and division (by reduction) of unit fractions (118.1–126.29); two methods of extraction of an approximate square root, the one a refinement of the other (128.1–134.22); an Easter Computus, assuming 1341 as current year (134.23–138.28)[21]; a so-called Μέθοδος πολιτικῶν λογαρισμῶν *Procedure of Civil Life Calculations*, namely: an exposition of the several species of the rule of three (140.1–144.9); generalities

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[16] Edition TANNERY, Notice 86–110, 114. The main manuscript witnesses are organized as follows: Vat. gr. 1411, ff. 10r–13r; its apographs are El Escorial, Real Biblioteca del Monasterio de S. Lorenzo, Φ.I.10 (*Diktyon* 15142), ff. 84r–90v (1542), an immediate copy of which is Par. gr. 2428 (*Diktyon* 52060), ff. 194r–202v (mid-16th century), copies of which (by the renowned scholar A. J. H. Vincent) are Par. suppl. gr. 819 (*Diktyon* 53520) (19th century), ff. 128–153, and the entire Par. suppl. gr. 820 (*Diktyon* 53521) (19th century); earlier copies of Vat. gr. 1411 are Vat. Ross. 986 (*Diktyon* 66453) (mid-15th century), ff. 142r–148v; Par. suppl. gr. 652 (*Diktyon* 53387), ff. 154v–160r (15th century). On all these manuscripts see ACERBI, Problemi. The treatise is also present, anonymous (title Παράδοσις σύντομος καὶ σαφεστάτη τῆς ψηφηφορικῆς ἐπιστήμης) and without the tables, in Marc. gr. Z. 323 (*Diktyon* 69794), ff. 9r–13v (beginning 15th century), and Vat. gr. 1058 (*Diktyon* 67689), ff. 84r–86v (same copyist as the Venice codex), and, preceded by the tables, in Par. Coislin 338 (*Diktyon* 49479) ff. 1r–8v (15th century). Tannery did not use these three manuscripts in his edition. He did not know of Vat. Chis. R.IV.20 (*Diktyon* 65207), ff. 183v–186v, either, to be dated 1360–80 and thereby constituting the oldest witness of Rhabdas' treatise. This is a slightly debased version, without the tables, of the *First Letter*, which, quite surprisingly, retains a designation used in *Anonymus* B (see note 113 below); this, and the other variant readings, might even corroborate the hypothesis (on which we shall not dwell) that the Chigi manuscript carries an inaccurate copy of a redaction of Rhabdas' treatise a subsequent revision of which emerges in Vat. gr. 1411. As for the other witnesses listed in the *pinakes* database, Par. gr. 3100 (*Diktyon* 52745) does not contain the treatise; the manuscript Cambridge, University Library, Kk.V.26 (gr. 2068) (*Diktyon* 12209), has it on ff. 44(3?)–51 (16th century); the manuscript Oxford, Bodleian Library, Holkham gr. 30 (*Diktyon* 48098), ff. 227r–228v (16th century), has only the section on finger-notation. The New Haven manuscript is a very late transcription.

[17] This is the only such description surviving in Greek. The oldest source on finger-notation is the section *De computo uel loquela digitorum* in Bede's (d. 735) *De temporum ratione* (namely, an Easter Computus), in *CCSL* 123B, 268–273 = *PL* 90, 689–693.

[18] Throughout this article, the noun phrase "numerical order" designates any of the monadic numbers, decads, hundreds, thousands, myriads, and so on. As we shall see on page 31, standard numerical tables normally operate on numerical orders.

[19] They carry the title ψηφοφορικόν· εὕρεμα Παλαμήδους "computational ‹set-out›: Palamedes' discovery"; they were only partly edited in TANNERY, Notice 110–116. Rhabdas refers to them at the end of the section on subtraction (ibid., 96.25–27). Tables with instructions of use partly identical to those in Rhabdas' *First Letter* can also be found in Marc. gr. Z. 323, ff. 25r–37v, and Vat. gr. 1058, ff. 33r–40r. Similar tables make the Ψηφάριον *Counting Booklet* comprising the entire Vat. gr. 1550 (*Diktyon* 68181) (14th century); they are edited in J. L. HEIBERG, Kleine Anecdota zur byzantinischen Mathematik. *Zeitschrift für Mathematik und Physik. Historisch-literarische Abtheilung* 33 (1888) 161–170: 165–170.

[20] Edition TANNERY, Notice 118–186, but two problems at the end are omitted because they were already published (on the basis of the ms. Zeitz, Stiftsbibliothek 67 (*Diktyon* 72776) [<John Argyropoulos>], ff. 97v–98r) in R. HOCHE (ed.), Nicomachi Geraseni pythagorei Introductionis Arithmeticae libri II. Lipsiae 1866, 152.4–154.10. The main manuscript witnesses are the same as for the *First Letter*, with the addition of Par. suppl. gr. 682 (*Diktyon* 53417), f. 34r–v (15th century), containing only the worked-out example of the Easter Computus (TANNERY, Notice 136.24–138.28). See TANNERY, Moschopoulos et Rhabdas 12–14, for a summary description of the contents of the treatise. On the Easter Computus see O. SCHISSEL, Die Osterrechnung des Nikolaos Artabasdos Rhabdas. *Byzantinisch-neugriechische Jahrbücher* 14 (1938) 43–59.

[21] The actual date is a.m. 6849, fitting the assumed date of Easter (April 8) and the other calendrical data.

and some problems of conversion involving weight[22], measure, and currency units of measurement, solved by application of the previous rules (144.10–154.5); the same for a problem involving alloying (154.6–24); twenty *Rechenbuch*-style problems[23], with solutions and associated procedures (156.25–186.19). As is apparent even from this summary, the contents of the *Letter to Tzavoukhes* are less homogeneous than those of the *Letter to Khatzykes*. In both *Letters*, Greek numerals are used.

ANONYMUS B IN BARB. GR. 4 AND IN OTHER MANUSCRIPT WITNESSES

The manuscript Barb. gr. 4 is a very small (128×85 mm) codex of III + 188 leaves (= 1–187 + 186[a]) in oriental paper. It contains a complex series of excerpta from grammarians, metricists, philosophers, and, most importantly, poets and tragedians; these extracts count as witnesses of some importance in the editions of almost all excerpted texts[24]. The three final quires, ff. 160–186, feature in particular Planudes' Greek translation of the so-called *Disticha Catonis* (ff. 160r–167r)[25], the Pythagorean *Carmen Aureum* (ff. 167r–168v)[26], a poem by Theodoros Prodromos (ff. 168v–169r)[27], twenty-two epigrams of the *Palatine Anthology* (in the margins of f. 167v and on ff. 169r–170v), and finally *Anonymus* B (ff. 171r–186v), which occupies exactly two quires. These last three quires have the following structure: ff. 160–170 (a senion lacking leaf 5); ff. 171–179 (a quinion lacking leaf 7); ff. 180–186 (a quaternion lacking leaf 6)[28]. As we shall see, this quaternion was originally a quinion that has lost its most external bifolium[29].

The composition of the codex is as follows—a triple vertical line marks a quire boundary characterized by the beginning of a fresh textual unit *and* by a variation of hand or of *ductus* of the same hand (both pointing to a non-sequential transcription), a double vertical line is present when the last of these three conditions does not apply:

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[22] The metrological portion at Tannery, Notice 144.11–146.8, is reprinted in E. Schilbach, Byzantinische metrologische Quellen (*Βυζαντινά Κείμενα και Μελέται* 19). Thessalonike 1982, 135–136; see also ibid., 30–31.

[23] Some of these problems coincide with problems edited in K. Vogel, Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts (*Wiener Byzantinische Studien* 6). Vienna 1968: no. 13 = example at Tannery, Notice 142.26–144.9; no. 14 = Rhabdas' problem I; 18 = problem III; 20 = IV; 21 = VI; 22 = VII; 9 = X; 11 = XII; 24 = XIII; 35 = XVI. Algebraic formulations of the problems in this section are in Tannery, Moschopoulos et Rhabdas 14. The entire Μέθοδος πολιτικῶν λογαρισμῶν is in fact a *Rechenbuch*: this is a collection of computational techniques and of arithmetical or metrological problems unrelated to each other, sometimes in (fictitious) daily-life guise. As a matter of fact, the "mathematical content" of some *Rechenbuch*-problems can be undressed and rewritten as Diophantine problems. On Byzantine *Rechenbücher* see F. Acerbi, Byzantine Rechenbücher: An Overview, with an Edition of Anonymi L and J. *JÖB* 69 (2019) in print.

[24] For Pindar (ff. 56r–64v), see J. Irigoin, Histoire du texte de Pindare. Paris 1952, 139–141 and 247–269; for Euripides, see K. Matthiessen, Exzerpte aus Sieben Tragödien des Euripides im Codex Vaticanus Barberini Graecus 4. *Hermes* 93 (1965) 148–158 (only the excerpts at ff. 9v–18r and 26r–32v); for Theocritus (ff. 72v–81v), see C. Gallavotti (ed.), Theocritus quique feruntur Bucolici Graeci. Roma 1946, 254; for Oppianus, *Halieutica* (ff. 22r–26r), see A. Zumbo, Gli Halieutika di Oppiano nella tradizione gnomologica. *RSBN* 34 (1997) 77–81; for the *Septem Sapientium Sententiae* (ff. 152r14–156v7), see M. Tziatzi-Papagianni, Die Sprüche der sieben Weisen. Zwei byzantinische Sammlungen. Stuttgart–Leipzig 1994, 11–21 and 448–450; for the epigrams, see below.

[25] On the manuscript tradition of this writing see V. Ortoleva, Massimo Planude e i Disticha Catonis. *Sileno* 15 (1989) 105–136; the edition is V. Ortoleva, Disticha Catonis in Graecum translata. Rome 1992.

[26] The Barberini codex is not mentioned in E. Diehl – D. Young (eds.), Theognis, ps.-Pythagoras, ps.-Phocilides, Chares, Anonymi Aulodia, Fragmentum Teliambicum. Lipsiae 1971, or in F. W. Köhler (ed.), Hieroclis in aureum Pythagoreorum carmen commentarius. Stutgardiae 1974.

[27] Edited in W. Hörandner, Visuelle Poesie in Byzanz. Versuch einer Bestandsaufnahme. *JÖB* 40 (1990) 1–42: 30–32.

[28] In all cases, no portion of text is lost because of this structure: the quires were purposely assembled in this way.

[29] See item 20 on pages 28–29 for details.

quire no.	1	2	3	4	5	6	7	8	9	10	11	12
quire type	IV–3	IV	IV	IV–2	IV	IV	IV	IV	IV	IV	IV	IV
last f.	5	13	21	27	35	43	51	59	67	75	83	91
quire sign.				α′	β′	γ′	δ′	ε′	ς′	ζ′	η′	θ′
hand	*a*	*a*	*ar*	*abc*	*ac*	*a*	*a*	*ac*	*ca*	*a*	*a*	*a*

quire no.	13	14	15	16	17	18	19	20	21	22	23	
quire type	IV	IV	IV	IV	IV	V–1	V–1	V	VI–1	V–1	V–3	1
last f.	99	107	115	123	131	140	149	159	170	179	186	186a
quire sign.	ι′	ια′	ιβ′	ιγ′	ιδ′							
hand	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*ar*	*a*	*a*	*a*	*a*

Capocci's distribution of the hands must be corrected[30]. A main copyist penned the entire volume. A second hand, very similar to the main hand and therefore contemporary to it, subsequently added some texts; these additions may occupy the central space of the page, whenever it had been left blank. Note that dubious attributions of some pages remain.

The main hand of the codex displays a script typical of the small galaxy of Planudes' students active in Constantinople during the first quarter of the 14th century[31]. These hands share the main features of the master's handwriting to variable degrees. They are quick, upright or leaned, small or medium-sized; they use a high variety of letter shapes, abbreviations and ligatures; the modular contrast typical of 13th-century cursive scripts is altogether absent. These post-Planudean hands are usually not easy to read (even for contemporaries); therefore, they were used, as Planudes was also accustomed to, to copy texts intended for non-commercial use within restricted circles. However, the round and pleasing handwriting of Barb. gr. 4 is skilled and professional enough to allow for a commercial use of the codex' ambitious selection of texts. In turn, the codex' small size suggests that its readers availed themselves of it for their leisure.

Barb. gr. 4 was probably designed to gather prominent excerpts from a literary canon available and broadly studied in the period in a small book, supplemented by some grammatical texts and by the short arithmetical tract we study in these pages. Planudes' (and his students') wide-ranging interests would fittingly provide enough reasons for the presence of an elementary mathematical treatise at the very end of a codex otherwise decidedly oriented towards grammar and poetry. Note, however, that the contents and the sectional nature of the last three quires seem to mark a slight departure from the principles underlying the collection of the preceding material.

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[30] According to V. Capocci, Codices Barberiniani Graeci. Tomus I. Codices 1–163. Città del Vaticano 1958, 6, hand *a* penned ff. 1r–18r6, 22r–143r, 160r–186v, and 186a; hand *b* ff. 18r7–21v, 152r14–159v, and marginal additions at ff. 28r and 30r; hand *c* ff. 143v–152r13 (this segment coincides with an anonymous rhetorical treatise). J. Irigoin (Histoire 247 n. 5), apparently referring to ff. 56–64, asserted that "le type d'écriture, assez carrée, et celui du papier, un bombycin à pontuseaux triples, nous permettent d'attribuer ce manuscrit à la seconde moitié du xiii^e siècle". D. Harlfinger (*apud* M. Tziatzi-Papagianni, Sprüche 11–12) likens the script of hand *a* to that of George Gemistos Plethon (d. 26/6/1452; *PLP*, no. 3630). Of course, this must not be taken as an indication that Barb. gr. 4 must be dated to the 15th century. The anonymous rhetorical treatise at ff. 143v–152r13 of Barb. gr. 4 is wrongly ascribed to Plethon in Par. gr. 2926 (*Diktyon* 52565), ff. 287v–291r (and as a consequence in Ch. Walz, Rhetores graeci. I–IX. Stuttgart–Tübingen–London–Paris 1832–36, VI 546–598), penned in the second half of the 15th century (*RGK* II, no. 72).

[31] C. Gallavotti (Planudea. *Bollettino del Comitato per la preparazione dell'edizione nazionale dei classici greci e latini*, n.s., 7 [1959] 25–50: 48–50) pointed out that the epigrams at ff. 167v and 169r–170v were copied from the so-called "silloge laurenziana" contained in ff. 3r–6v and 381v–384r of Laur. Plut. 32.16 (*Diktyon* 16280), the celebrated collection of hexametric poetry realized in 1280–83 within Planudes' atelier. This sets a *terminus post quem* to the transcription of Barb. gr. 4. On some of these epigrams and for literature on Laur. Plut. 32.16, see most recently F. Valerio, Planudeum. *JÖB* 61 (2011) 229–236. The "silloge laurenziana" was penned by Planudes himself.

Even if the volume was a clear and neat product, the copyist had no qualms in filling the margins with supplementary texts, creating in most pages an impression of overcrowding. However, the organization of the contents is made clear through the use of rubricated headbands, initials and titles in the margin[32], a procedure that would have eased the elaboration of a table of contents that was never redacted or that the volume no longer has in its present form[33].

The hand of the main copyist (hand *a*) can be found on ff. 1r–18r6, 18v–19v5 a.i., 21r–25r, 26r6, 29r, 30v–56r, 64v–152r13, 160r–186v. Its general appearance may change, as happens on ff. 143v–152r, which Capocci attributed to a different hand and where the handwriting of copyist *a* is of a slightly greater size and more solemn[34].

The main copyist's handwriting is characterized by the following features:

- frequent use of majuscule *alpha*
- almost general absence of majuscule *beta* and *delta* and of minuscule *gamma*
- the elegant ligatures ελ and ελλ
- the ligature εξ, tall and mostly shaping a right angle at the top
- slim *zeta* and *ksi*
- open *theta* starting from the base line without the usual initial curve
- the group ου, written in ligature upon the text line, in the middle of a world, not as an ending
- the group συν and the similar ligatures σα, σο, σω, featuring a long C-shaped *sigma*
- the frequent ligature of *omega* with its circumflex accent
- the large abbreviation signs of μ(εν), ον, etc. in the shape of a crescent moon
- the abbreviation of καί with the first two strokes shaping a L

This round and agile script may be compared with those of copyist C of Par. gr. 1040 (*Diktyon* 50633), dated from 1325[35], of the monk Kassianos who copied Planudes' writings and belonged to Gregoras' circle at the Chora[36], or of Manuel Gabalas (Matthew of Ephesos)[37].

Two further copyists (hands *b* and *c*) transcribed ff. 25v–26r5 and ff. 26r7–28v, 29v–30r, 56v–64r[38], respectively. As for hand *b*, a wider spacing of the letters, a closed *theta* with an elongated crossbar, and an upright *epsilon* suggest it should not be identified with *a*. As for hand *c*, differences with respect to *a* include a swollen *alpha*, *omicron*, *sigma*, and *ypsilon*, majuscule *beta*, *my* with thick vertical strokes, a closed long *omega* made by a horizontal stroke and two loops. These letters differentiate handwriting *c* from that of the main copyist; nevertheless, on ff. 56v–64r such differences are less prominent, casting doubts on the individuality of the hand.

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[32] The copyist draws simple but not clumsy headbands at the beginning of the main texts. On f. 113r he makes an attempt at writing in epigraphic capital letters.

[33] The last f. 186a should be placed before f. 1; since the resulting first quire has six folios (186a + 1–5), the table of contents might have been located in the first two folios of the volume.

[34] As previously stated, these pages contain the Συντομὴ περί τινων μερῶν τῆς ῥητορικῆς published by Walz under the name of George Gemistos Plethon. A fact corroborating the contention that this text was copied at a different time is that a darker ink is used, also for the initials. Other leaves such as ff. 18v–19v display a similar *ductus*.

[35] P. Géhin, Les manuscrits grecs datés des XIII[e] et XIV[e] siècles conservés dans les bibliothèques publiques de France. Vol. 2. Première moitié du XIV[e] siècle (*Monumenta palaeographica Medii Aevi. Series Graeca* 1). Paris–Turnhout 2005, 34–36 and pl. 25.

[36] *RGK* III, no. 353 (wrongly dated to the second half of the 14th century); I. Pérez Martín, El scriptorium de Cora: un modelo de acercamiento a los centros de copia bizantinos, in: Ἐπίγειος οὐρανός. El cielo en la tierra. Estudios sobre el monasterio bizantino, ed. P. Bádenas – A. Bravo – I. Pérez Martín (*Nueva Roma* 3). Madrid 1997, 203–223: 220–221.

[37] *PLP*, no. 3309 (b. 1271–2, d. 1355–60); *RGK* I, no. 270; II, no. 370; III, no. 445.

[38] This section of the manuscript does not contain a sharply distinguished set of excerpts.

Subsequently to the work of the main copyist, a second hand (*r*)[39] added some texts in the margins or in blank pages, using two different inks. An ochre ink is used on ff. 18r7–17r, 19v5 a.i.–20v5, bottom margin of f. 30r, upper margin of f. 36v, interlinear space of f. 52v, ff. 152r14–159v (end of quire 20), and bottom margin of ff. 160r and 166v. With common brown ink, *r* penned the text on f. 20v, from line 6 as far as the end, after his own ochre-colored addition, and the two lines in the bottom margin of f. 21r; the texts in the bottom margin of ff. 3v, 9v, 11v, side margin of f. 25v, bottom margin of f. 28r, side margin of ff. 30v, 32r, 35r, bottom margin of f. 56r, bottom and side margin of f. 163v. In *Anonymus* B, its interventions, in brown ink, can be found in the interlinear space of ff. 173v, 177v, 179r, in the margins of ff. 174r, 176r–v, 177v, 178v, as corrections to the text in ff. 176r–v, 177r–v. This handwriting shares some features with that of copyist *a*; still, it looks like as a simplified, edgy, and rough version: it consistently spaces letters more than copyist *a* does and uses slightly different letter shapes (see for instance *theta* and *ksi*) as well as majuscule *delta*. We may thus safely regard this reviser as contemporary to the copy, perhaps the person who commissioned it or who shared its use shortly it had been copied.

We shall see in note 110 that Rhabdas' *First Letter* seems to retain traces of the reviser's correction on f. 177v. One might thus be tempted to identify the reviser with Rhabdas himself. It is also true, on the other hand, that the character of the reviser's main additions to the collection in Barb. gr. 4 attests for interests not confirmed by the extant documentary record on Rhabdas. The main texts transcribed by copyist *r* include the entire set of extracts from philosophical writings: Plato at f. 18r7–17; Epictetus and Heraklitos at 19v5 a.i.–20v5; a consistent set of excerpts dealing with sapiential lore at 152r14–159v.

The contents of Barb. gr. 4 are described in detail in V. Capocci's catalogue[40]. However, he could not identify the treatise at ff. 171–186 as a version of Rhabdas' *First Letter*. Therefore, he simply recorded its title, *incipit*, and *desinit*. The latter is found at f. 179r since, as Rhabdas' *First Letter* does, *Anonymus* B ends with a series of tables. As the treatise is located at the end of the manuscript, it has suffered from damage typically arising in this position: f. 186v is severely faded; moreover, we shall see that the last quire has lost its most external bifolium, to be located before f. 180 and after f. 186[41].

During his *iter italicum* of 1886 and after publishing his edition of Rhabdas' *Letters*[42], Tannery discovered a further witness of *Anonymus* B: Vat. gr. 1481 (*Diktyon* 68112), ff. 180r–201v, copied at the beginning of 17th century by John Santamaura[43]. Tannery briefly described the treatise but satisfied himself with asserting that it was a new witness of Rhabdas' *First Letter*. Still, he should have noticed that the text was remarkably different from the one he himself had published; maybe he was deluded by his own assessment of this version of the text as a revision (see just below). Certainly, he had to work in a hurry, checking hundreds of manuscripts in one month and travelling between Turin, Milan, Venice, Rome, and Naples.

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[39] There are additions by a third, elegant hand using black ink; they are in the upper margin of f. 5v, in the side margin of f. 113r, and in the upper margin of ff. 132r, 133r, 135r, 138r, 140v–141r.

[40] See also the detailed description, with bibliography, at https://digi.vatlib.it/view/MSS_Barb.gr.4, where a reproduction of the manuscript can be accessed.

[41] This loss was not noted by Capocci.

[42] P. Tannery, Rapport sur une mission en Italie du 24 Janvier au 24 Février 1886. *Archives et Missions scientifiques et littéraires*, 3e série, 13 (1888) 409–455, repr. Id., Mémoires Scientifiques II. Toulouse–Paris 1912, 269–331: 318.

[43] Self-ascription on f. 190v, see page 12 below. On Santamaura, see *RGK* I, no. 179; II, no. 238; III, no. 299; M. D'Agostino, Giovanni Santamaura. Gli ultimi bagliori dell'attività scrittoria dei greci in Occidente. Cremona 2013. None of these reference works mentions Vat. gr. 1481.

We know in fact of another, very partial, witness of *Anonymus* B: f. 47r–v of the composite 16th-century manuscript Par. gr. 2535 (*Diktyon* 52167)[44]. It contains the very beginning of the treatise, partly collated with a manuscript containing Rhabdas' *First Letter*; the transcription breaks off in the middle of a sentence[45]. Tannery availed himself of this manuscript in his edition; on the basis of such scanty evidence, he could only regard the text in Par. gr. 2535 as a "recension spéciale", "d'une date relativement récente sans aucun doute"[46]. Tannery also remarked that this text was noteworthy because of the "suppression" of a conspicuous part of the preface, amounting in fact to a long extract from the introduction of Diophantos' *Arithmetica* and repeated verbatim in both of Rhabdas' *Letters*.

As we shall see, there is not the slightest doubt that Santamaura used Barb. gr. 4 as his model. We might press the story of this transcription a little further. In 1614, F. Morel published a plaquette containing two texts on the representation of numbers on the fingers of the hands. The two texts are the section *De computo uel loquela digitorum* in Bede's *De temporum ratione*, and the dedicated section in Rhabdas' treatise. At least, this was what Morel and Tannery believed, and rightly so on the basis of the information they had: Morel declares that he used a collation by Lelio Ruini, a renowned book collector and then Apostolic Nuncio to Poland[47]; Ruini had in his turn used a unspecified *codex Vaticanus*. Now, on the one hand, Morel expressly declares that the extract comes from a treatise by Rhabdas; on the other hand, the variant readings involved unquestionably show that Morel's text coincides with *Anonymus* B. To Tannery, thus, what Morel published was a fragment of the *First Letter*, "d'après une copie du manuscrit 1411 du Vatican"[48], namely, a part of the debased version that had already surfaced in Par. gr. 2535. As Santamaura customarily copied for Ruini, we might surmise that, on the occasion of Morel's request, Santamaura first achieved a complete copy of *Anonymus* B, extracting then the part on finger-notation to be sent to Morel. This hypothesis is refuted by a complete collation of Santamaura's copy in Vat. gr. 1481: it bristles with mistakes, including several *saut du même au même*; no conjunctive variant readings with Morel's edition have been found.

Thus, things are less simple than Tannery might have supposed. It might even be that Ruini used a manuscript containing *Anonymus* B different from Barb. gr. 4 but ascribing this version to Rhabdas. This manuscript, if any ever existed, has escaped our systematic searches in the catalogues[49]. However, we must stress that this is not necessarily the case, since Morel obviously knew of the version of the same treatise expressly ascribed to Rhabdas: as he himself asserts, a mathematician and friend of his had uncovered both *Letters* in a manuscript of the Bibliothèque Royale in Paris and it was for precisely this reason that he had renounced publishing the entire treatise.

EDITION OF ANONYMUS B

We first edit the treatise, passing then to its analysis. As will be apparent from the edition, the transcription of *Anonymus* B in ff. 171–186 of Barb. gr. 4 is almost faultless; we may take this as a sign that it is not much removed from the original. The text has marginal titles in red, mostly written in

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44 On this manuscript, see T. J. Mathiesen, Ancient Greek Music Theory. A Catalogue Raisonné of Manuscripts (*International Inventory of Musical Sources* B 11). Munich 1988, 245–248.

45 The text runs as far as Tannery, Notice 88.18 γραμμῆς. The last three words constitute a *reclamans*. No variants allow to identify the collation manuscript.

46 Ibid., 83.

47 F. Morel, Nic. Smyrnaei Artabasdae Graeci mathematici ἔκφρασις Numerorum Notationis per gestum digitorum … Lutetiae 1614, 3–4, and Tannery, Notice 74–75 and 80.

48 Recall that Vat. gr. 1411 is the main witness of the *First Letter*. Tannery did not know of it when he published his edition of Rhabdas' *Letters*.

49 Barb. gr. 4 is declared to be a belonging of the San Salvatore convent in Venice, there bequeathed by some *dominus de fosset* (f. 8r, a hand of 16th century), then (the date is 1734) to the Badia of Grottaferrata, under no. 295.

vertical, rubricated letters at the beginning of self-contained sections, and an initial decoration; the frames of the numerical tables at ff. 179v–186v are in red, as well as some leading numeral letters in the addition and subtraction table and some of the inscriptions contained in the tables themselves. Within the text, the initial list of numeral signs is in red. The main copyist is responsible for all of this. As previously stated, the text is also corrected by a hand different from that of the main copyist: this reviser integrates, in the margins of ff. 174r, 176v, 178v, some passages omitted in the transcription, and at f. 177v he appears to correct the text. The margins of ff. 171r–172v and 175v–176r contain short annotations that do not have anything to do with *Anonymus* B[50].

At f. 172r, a rough representation of a human hand partly illustrates the section on finger-notation[51]; the hand is accompanied by some short inscriptions, penned in red ink, indicating the names of the fingers: μύωψ μικρός or little finger; ἐπιβάτης καὶ παράμεσος or ring finger; σφάκηλος μέσος or middle finger; λιχανός or forefinger, ἀντίχειρ or thumb. The only name written within the diagram in the area of the hand's palm is the designation κύαθος. The digits from α to ε are also inscribed at the base of the fingers, starting as usual from the little finger[52]. The school practice of representing numbers on the fingers of a hand is witnessed for Constantinople by Nicholas Mesarites' *Description of the Church of the Holy Apostles*, written between 1198 and 1203[53].

The text of *Anonymus* B is edited on the basis of Barb. gr. 4; where the manuscript is now illegible[54], we resort to the readings of Vat. gr. 1481. That this manuscript is a (quite debased) direct copy of the Barberini codex is borne out by the following facts.

Vat. gr. 1481 is a composite manuscript; our interest will only be focused on ff. 180r–201v + 181a, 191a. Santamaura himself numbered the first nineteen pages of this textual unit, in the upper external corner and from 1 to 19. These numbers were partly deleted and replaced by the present folio num-

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50 The additions at ff. 171–172 are apparently lexicographical material. As seen, relevant texts inscribed in the margins are also found elsewhere in this manuscript.

51 Sometimes inappropriately called "finger-reckoning": as a matter of fact, no computation is performed.

52 This illustration does not fit the description provided in the text: in the latter, numbers are assigned to specific and non-cumulative configurations of all fingers of one hand, not to single fingers—and the configurations representing the numbers from 1 to 5 (in which only three fingers are used) cannot in any way fit an assignment of digits from 1 to 5 like the one in the hand depicted in the Barberini codex. Thus, the functionality of the diagram is very limited. It is only useful insofar as, first, it instructs the reader about what each of the fingers is called and second, it indicates the succession in which the fingers are dealt with throughout the text, namely, starting from the little finger and proceeding towards the thumb. In other words, the diagram would rather suit the lexicographical needs of the reader, by extracting relevant vocabulary, than aid his or her knowledge of finger-notation. Nevertheless, the inclusion of the diagram is worth noting as a visual aid. Among the available copies of either *Anonymus* B or Rhabdas' *First Letter*, such a *depictio manus* is only preserved in Barb. gr. 4 and in Vat. gr. 1481. By contrast, the *De computo uel loquela digitorum* in Bede's *De temporum ratione* occasionally features hand diagrams and representations, as for instance in the 9th-century Vat. Pal. lat. 1449, f. 118v. In general, while there is a relatively rich tradition of hand diagrams used as mnemonic devices in the Latin West (both in the context of finger-notation and of the study of music), little is known of the Byzantine equivalent if such tradition indeed existed. A sample of late Byzantine depictions of cheironomic gestures (indicating intervals and melodic figures) is discussed in N. K. Moran, Singers in Late Byzantine and Slavonic Painting. Leiden 1986.

53 This is ch. 10.1 of Mesarites' writing: see G. Downey, Nikolaos Mesarites: Description of the Church of the Holy Apostles at Constantinople (*Transactions of the American Philosophical Society* 47.6). Philadelphia 1957, 866 (transl.) and 899 (text). On the other hand, the recurrent use, in logistic and astronomical texts, *Rechenbücher*, and Easter Computi, of κρατεῖν for "keeping" a number in order to use it in a subsequent operation, does not imply anything as to a possible application of finger-notation. Even if the verb is sometimes qualified by expressions like ἐν χερσί "in your hands" (see for instance the Easter Computus in Matthew Blastares' Σύνταγμα, 414–415 ed. G. A. Rhalle – M. Potle, Σύνταγμα τῶν θείων καὶ ἱερῶν κανόνων τῶν τε ἁγίων καὶ πανευφήμων ἀποστόλων, καὶ τῶν ἱερῶν οἰκουμενικῶν καὶ τοπικῶν συνόδων, καὶ τῶν κατὰ μέρος ἁγίων πατέρων, VI, Athens 1859, 31–518; we thank O. Delouis for pointing out this passage to us), and we must think that such a qualification is understood in every instance, the point is that it is simply impossible to represent a 4-digit number, as for instance an a.m. year date is, on the fingers of two hands only!—actually, this cannot even be achieved for 2-digit numbers made of decads and units.

54 The restoration of the codex makes it impossible to use Wood's lamp.

bers. The first seven folios, ff. 180–185 (+ 181a) are a quaternion lacking leaf 6; the remaining folios, most of which are glued on reinforcement flaps, are not organized in a consistent quire structure. The bottom margin of f. 181v contains a *depictio manus* whose lower half has been cut off. One reads the digits γ δ ε on the fingers' bases; the inscriptions are κύαθος in the palm and, from bottom to top and in front of each fingertip: *cut off fingers*, παράμεσον καὶ ἐπιβάτης, σφάκηλος καὶ μέσος; λιχανός, ἀντίχειρ. On f. 181ar, a second *depictio manus* is present, traced on a paper rectangle glued on the page. No digits are marked on the fingers' bases; the inscriptions are κύαθος and, from bottom to top and in front of each fingertip, μύωψ, παράμεσος ὁ καὶ ἐπιβάτης, σφάκηλος ὁ καὶ μέσος, λιχανός, ἀντίχειρ. Folio 181av is blank.

Santamaura inserts all the corrections by copyist *r* in his transcription; see in particular, in our apparatus to the text, the difficulties he has in reading the set of interventions in Barb. gr. 4, f. 177v. Quite characteristically, Santamaura frequently comments on some features of his model.

On f. 189v, a first version of the table of numerals πλινθὶς ἀρίστη τῆς τῶν ἀριθμητικῶν στοιχείων καταγραφῆς is traced and is presently set out in a topsy-turvy manner. The table is deleted by two pen strokes. In the external margin, one finds the Latin inscription *ad maiorem intelligentiam*, under which the numerals in the last column of the table are repeated, the tremas characterizing myriads of myriads (see item 19 on page 28) being more appropriately located. The same table can be found at f. 190r.

Santamaura realized that the beginning of the addition and subtraction table was missing in his model. Accordingly, on f. 190v we read the inscription ὁ νῦν βιβλιογράφος {s.l. rubro pictum Ἰω(άννης) ὁ Σαγκταμαύρ(ας) ἐστίν m.1} εὑρὼν ἐν τῷ παλαιῷ ἀντιγράφῳ, ἀφ᾽ οὗ τὸ παρὸν νέον ἀντίγραφον ἐξισοῦται, ἄρχεσθαι τὴν τῶν ἀριθμῶν σύνθεσιν ἀπὸ τοῦ τ ἑκατονταδικοῦ στοιχείου, ὅθεν ᾠήθη ἐλλειπὲς εἶναι τὸ ῥηθὲν παλαιὸν ἀντίγραφον, πρὸς οὖν [[την]] τελείαν συμπλήρωσιν τῆς συνθέσεως τῶν ἀριθμητικῶν στοιχείων κατέστρωσεν [[τὴν ὑποτεταγμένην]] οἴκοθεν τὰ ὑποτεταγμένα στοιχεῖα τῆς συνθέσεως, ἀρχόμενος ἀπὸ τῶν μονάδων καὶ δεκάδων ὁμοῦ καὶ μέρους τῶν ἑκατοντάδων, τοῦ ρ καὶ σ. At the beginning of the sequence of reconstructed tables, we read the inscription ἑκατονταδικῶν μορίων. Every table on ff. 191r–191ar is headed ὁ βιβλιογράφος; on f. 191r this is followed by ἀρχὴ τῆς συνθέσεως, in red ink. In a cell within the table on f. 191av, before the beginning of the addition subtable for τ (300), we read the inscription ἕως ὧδε τὰ τοῦ βιβλιογράφου, ἐντεῦθεν ἄρχεται τὰ τοῦ παλαιοῦ ἀντιγράφου.

Santamaura thought that the table of addition and subtraction had to be completed. Thus, before the end of the table at f. 192v, in two consecutive cells within the table, he writes ἄχρι τοῦδε τὰ ἐκ τοῦ παλαιοῦ ἀντιγράφου and ἐντεῦθεν ὁ βιβλιογράφος ἄρχεται καὶ περὶ τῶν μυριάδων. The subsequent ff. 192v–194v set out an addition and subtraction table for myriads, absolutely identical to the previous table apart from the standard presence of tremas denoting myriads. Every table on ff. 192v–194v is headed ὁ βιβλιογράφος; before the end of the table on f. 194v, in three consecutive cells within the table, inscriptions ἕως ὧδε τὰ τοῦ βιβλιογράφου || *ex uetero exemplari* τέλος συνθέσεως καὶ ἀφαιρέσεως || *librarius uide in pagina 9* περὶ ἐκβολῆς ἤτοι ἀφαιρέσεως· καὶ ἀρκεῖ.

In the multiplication table, Santamaura corrects a repeated numerical order mistake of Barb. gr. 4; he justifies his corrections by marking a red cross by the side of the faulty numerals and by writing in the external margin ὁ βιβλιογράφος *corrig.* +ιβ *pro* ι͵β || +ις *pro* ι͵ς || +κδ *pro* κ͵δ || +κη *pro* κ͵η || +λβ *pro* λ͵β || +λς *pro* λ͵ς, where each digit not preceded by the lower left stroke carries a superimposed trema. On ff. 200v and 201r, corrections are effected by the marginal annotations *librarius* υπ *pro* υη || φξ *pro* φς || χμ *pro* χδ || ψκ *pro* ψβ ||; *librarius* ͵βρ *pro* ͵αρ; and *librarius* ͵γξ *pro* ͵γλ, each digit carrying a superimposed trema.

Most crucially, the last table contains, within a cell at the end of the first column, the inscription *desiderantur reliqua apud exemplare* [[*an*]] *antiquo desunt folii*; and at the beginning of the second column, ὁ δὲ διπλοῦς καὶ τριπλοῦς καὶ ἐπέκεινα πολλαπλασιασμὸς ἀεὶ [[ψαφι]] γίνεται οὕτως. ψαφισάτω τὸ α μετὰ τοῦ α, τὸ β μετὰ τοῦ β, εἶτα καὶ εἰς γ. τὸ α μετὰ τοῦ ἁπλοῦ α, μετὰ τοῦ β καὶ τὸ β *et reliqua desunt*. We shall see that the last words, which, unlike us, Santamaura was able to read, provide a crucial piece of information.

All of this exactly fits the present status of Barb. gr. 4. For this reason, we shall not discuss the readings of Santamaura's transcription; we report in the apparatus those of his readings which help in deciphering difficult passages of Barb. gr. 4. We also resort once to Par. gr. 2535. Morel's text does not present any relevant readings.

Our edition normalizes the punctuation: in a technical treatise, there is really no point in adhering to Byzantine conventions in such matters. We adopt a "light" punctuation; in particular, short-range correlatives μὲν … δὲ are not separated by a comma. The text in Barb. gr. 4 employs the *dicolon* for very strong pauses (frequently followed by a *paragraphos*; they are always followed by a conspicuous blank space and by a rubricated initial letter); upper point for strong pauses (sometimes followed by a blank space); lower point and comma (the latter quite frequently) for light pauses, and for separating units of meaning in a sentence—as is common in Byzantine punctuation—, the items of a list (the comma being subordinated to the lower point), and the result of an operation from the operands. We did not find examples of commas used as *diastolē*. Oxytone words are barytonized before a light pause. It is impossible to say whether the middle point is also used or not. In general, the copyist of Barb. gr. 4 is also parsimonious with punctuation. By contrast, we retain the original convention about the presence of movable *ny* and *sigma* and about the accentuation of enclitics. We put a paragraph and a capital letter whenever the manuscript has either a title in the margin or a rubricated initial letter; because of this convention, and since *all* titles of the sections of the treatise are located in the margins, we refrain from pointing out such characteristics of Barb. gr. 4 in our critical apparatus. We also publish the tables and regularize the use of tremas; a standard shortcut is put into effect in the tables: a single trema is superimposed to myriad numerals made of several digits; in principle, each such digit should instead receive the trema. A trema over a numeral letter is replaced by an apex in the present transcription; no apex is apposed to numeral letters as such. We generally do not correct the text of Barb. gr. 4, but record the expected or required reading in the apparatus.

Sigla:

B Città del Vaticano, Biblioteca Apostolica Vaticana, Barberinianus gr. 4
V Città del Vaticano, Biblioteca Apostolica Vaticana, Vaticanus gr. 1481
P Paris, Bibliothèque nationale de France, gr. 2535

|$_{171r}$ παράδοσις σύντομος καὶ σαφεστάτη τῆς ψηφηφορικῆς ἐπιστήμης ῥᾴστη τοῖς ἐθέλουσι ταύτην μετελθεῖν, ἥτις καὶ ἔχει οὕτως.

Δεῖ τὸν βουλόμενον μετελθεῖν τὴν τῶν ἀριθμῶν ἐπιστήμην τοῦτον τὸν τρόπον προχωρῆσαι· πρῶτον μὲν μαθεῖν πόσα στοιχεῖα εἰσὶ τὰ συμβαλλόμενα εἰς αὐτὴν καὶ πόσον ἀριθμὸν σημαίνει ἕκαστον αὐτῶν, εἶτα πῶς δεῖ τοὺς ἀριθμοὺς κρατεῖν ἐν ταῖς δυσὶ χερσί, μετὰ τοῦτο τὰ παρεπόμενα αὐτῇ διδαχθήσεσθαι, εἶτα ἑαυτὸν φέροντα δοῦναι τῷ τῆς ὑποθέσεως οἱονεὶ σώματι.

α. περὶ τῆς τῶν στοιχείων ἐκθέσεως

στοιχεῖα μὲν οὖν εἰσι τὰ δηλοῦντα τὴν ποσότητα καὶ τὸ μέτρον ἑνὸς ἑκάστου τῶν ἀριθμῶν ταῦτα, α β γ δ ε ϛ ζ η θ ι κ λ μ ν ξ ο π ϙ ρ σ τ υ φ χ ψ ω ϡ, καὶ τὸ μὲν α σημαίνει ἕν, τὸ β δύο, τὸ γ τρία, τὸ δ τέσσαρα, τὸ ε πέντε, τὸ ἐπίσημον ἕξ, τὸ ζ ἑπτά, τὸ η ὀκτώ, τὸ θ ἐννέα· ταῦτα μέχρις ὧδε μονάδας καλοῦμεν. πάλιν τὸ ι δηλοῖ δέκα, τὸ κ εἴκοσι, τὸ λ τριάκοντα, τὸ μ τεσσαράκοντα, |$_{171v}$ τὸ ν πεντήκοντα, τὸ ξ ἑξήκοντα, τὸ ο ἑβδομήκοντα, τὸ π ὀγδοήκοντα, τουτὶ τὸ σημεῖον[55] ϙ ἐνενήκοντα· ταῦτα μέχρι τοῦδε καλοῦμεν δεκάδας. καὶ αὖθις τὸ ρ ἑκατόν, τὸ σ διακόσια, τὸ τ τριακόσια, τὸ υ τετρακόσια, τὸ φ πεντακόσια, τὸ χ ἑξακόσια, τὸ ψ ἑπτακόσια, τὸ μέγα ω ὀκτακόσια, καὶ ὁ λεγόμενος ϡ χαρακτὴρ ἐννακόσια· τὰ τοιαῦτα δὲ ἑκατοντάδας προσαγορεύομεν. ταῦτα δὲ γραμμῆς[56] μὲν ὑπογραφομένης αὐτοῖς χιλιάδας δηλοῦσιν ὅσας μονάδας ἐδήλουν ἀπούσης τῆς γραμμῆς, δύο δὲ στιγμῶν ἐπιτιθεμένων μυριάδας. οἷον τὸ μὲν ͵α μετὰ γραμμῆς ἁπτομένης αὐτοῦ καὶ λοξῶς ἐπὶ τὰ ἀριστερὰ καταφερομένης δηλοῖ χιλιάδα μίαν, τὸ ͵β δύο, καὶ τὰ λοιπὰ δηλονότι τῶν στοιχείων οὕτω τὴν γραμμὴν δεξάμενα χιλιάδας δηλοῦσι τοσαύτας ὅσας ἐδήλουν μονάδας ἀπούσης τῆς γραμμῆς. καὶ πάλιν τὸ α χωρὶς τῆς γραμμῆς[57] ἐπιτεθεισῶν αὐτῷ δύο στιγμῶν μύρια δηλοῖ, καὶ τὸ β΄ ὁμοίως δύο μυριάδας, καὶ τὸ γ΄ τρεῖς, καὶ ἑξῆς ὁμοίως. εἰ δὲ καὶ παρούσης τῆς γραμμῆς ἐπίκεινται αἱ στιγμαί, τότε τὸ ὑποκείμενον στοιχεῖον μυριάδας δηλοῖ χιλιονταδικὰς τοσαύτας ὅσας χιλιάδας ἐδήλου μὴ παρουσῶν τῶν στιγμῶν. |$_{172r}$ εἰ δὲ ἐπάνω τῶν στιγμῶν ἕτεραι πάλιν τεθῶσι στιγμαί, δηλονότι μυριάκις ἐπιδίδωσι τὸ στοιχεῖον τὴν ἐνοῦσαν αὐτῷ ποσότητα, καὶ ἑτέρων ἔτι ἐπιτεθεισῶν στιγμῶν τὸ αὐτὸ ἀναλόγως συμβήσεται, καὶ ἔτι ἑτέρων, ἕως ἂν ὑπ' ἀπειρίας κωλύοιτό τις.

ἔκφρασις τοῦ δακτυλικοῦ μέτρου

Ἐν δὲ ταῖς χερσὶ καθέξεις τοὺς ἀριθμοὺς οὕτως. καὶ ἐν μὲν τῇ εὐωνύμῳ, ἀεὶ ὀφείλεις τὰς μονάδας καὶ δεκάδας κρατεῖν, ἐν δὲ τῇ δεξιᾷ τὰς ἑκατοντάδας καὶ χιλιοντάδας[58]· τοὺς δὲ ἐπέκεινα τούτων ἀριθμοὺς χαράττειν ἔν τινι· οὐ γὰρ ἔχεις ὅπως καθέξεις ἐν ταῖς χερσί. συστελλομένου τοῦ πρώτου καὶ μικροῦ δακτύλου – τοῦ μύωπος καλουμένου – τῶν δὲ τεττάρων ἐκτεταμένων καὶ ἱσταμένων ὀρθίως, κατέχεις ἐν μὲν τῇ ἀριστερᾷ χειρὶ μονάδα μίαν ἐν δὲ τῇ δεξιᾷ χιλιοντάδα μίαν. πάλιν συστελλομένου καὶ τούτου καὶ τοῦ μετ' αὐτὸν δευτέρου δακτύλου – τοῦ παραμέσου καὶ ἐπιβάτου καλουμένου – τῶν δὲ λοιπῶν τριῶν ὡς ἔφημεν ἡπλωμένων, κρατεῖς ἐν μὲν τῇ εὐωνύμῳ δύο ἐν δὲ τῇ δεξιᾷ ͵β. τοῦ δ' αὖ τρίτου συστελλομένου – ἤτοι τοῦ σφακέλου καὶ μέσου κειμένων – καὶ τῶν ἑτέρων δύο τῶν δὲ[59] |$_{172v}$ λοιπῶν δύο ἐκτεταμένων – τοῦ λιχανοῦ λέγω καὶ τοῦ ἀντίχειρος –, εἰσὶν ἅπερ κρατεῖς ἐν μὲν τῇ λαιᾷ γ ἐν δὲ τῇ δεξιᾷ ͵γ. πάλιν συστελλομένων τῶν δύο – τοῦ μέσου καὶ παραμέσου ἤγουν τοῦ δευτέρου καὶ τρίτου – καὶ τῶν ἄλλων ὄντων ἐξηπλωμένων – τοῦ ἀντίχειρος λέγω καὶ τοῦ λιχανοῦ καὶ τοῦ μύωπος –, εἰσὶν ἅπερ κρατεῖς ἐν μὲν τῇ λαιᾷ δ ἐν δὲ τῇ ἑτέρᾳ ͵δ. πάλιν τοῦ τρίτου τοῦ καὶ μέσου συνεσταλμένου καὶ τῶν λοιπῶν τεσσάρων ἐκτεταμένων, δηλοῦσιν ἅπερ κρατεῖς ε ἐν δὲ τῇ δεξιᾷ ͵ε. τοῦ ἐπιβάτου πάλιν τοῦ καὶ δευτέρου συνεσταλμένου καὶ τῶν λοιπῶν ἡπλωμένων, κρατεῖς ἐν μὲν τῇ εὐωνύμῳ ϛ ἐν δὲ θατέρᾳ ͵ϛ. τοῦ μύωπος πάλιν τοῦ καὶ πρώτου ἐκτεταμένου καὶ τῇ παλάμῃ προσψαύοντος τῶν δὲ λοιπῶν ἱσταμένων ὀρθίων, εἰσὶν ἅπερ κατέχεις ζ ἐν δὲ τῇ ἄλλῃ ͵ζ. τοῦ δευτέρου πάλιν τοῦ καὶ παραμέσου ὁμοίως ἐκτεταμένου καὶ κλίνοντος ἄχρις οὗ τῇ κυάθῳ τελείως προσεγγύσῃ κειμένου καὶ τοῦ πρώτου, τῶν δὲ λοιπῶν τριῶν – τοῦ τρίτου, τοῦ τετάρτου καὶ τοῦ πέμπτου, ὡς |$_{173r}$ προείρηται – ἱσταμένων ὀρθίων, τὸ γενόμενον σχῆμα ἐν μὲν τῇ λαιᾷ δηλοῖ η ἐν

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55 τουτὶ τὸ σημεῖον **P** : τουτὶ dein ἐν eras. et [[..]] τὸ σημεῖον add. s.l. m.1 **B** : τοῦτο ὅ ἐστι σημεῖον **V**

56 des. **P**

57 καὶ πάλιν τὸ α χωρὶς τῆς γραμμῆς s.l. m.1 **B** et teste **V**

58 καὶ χιλιοντάδας s.l. m.1 **B**

59 depictio manus marg. inf. **B**

δὲ τῇ δεξιᾷ ͵η. οὕτως οὖν καὶ τοῦ τρίτου ὁμοίως γινομένου, κειμένων καὶ τῶν ἄλλων δύο τοῦ πρώτου καὶ δευτέρου κατὰ τὸ αὐτὸ σχῆμα, ἐν μὲν τῇ ἀριστερᾷ δηλοῦσιν ἐννέα ἐν δὲ τῇ ἄλλῃ ͵θ. πάλιν τοῦ ἀντίχειρος ἡπλωμένου, οὐχὶ δ' ὑπεραιρομένου ἀλλὰ πλαγίως πῶς, καὶ τοῦ λιχανοῦ ὑποκλινομένου μέχρις ἂν τῷ τοῦ ἀντίχειρος προτέρῳ ἄρθρῳ συμπέσῃ ἕως ἂν γένηται σίγματος σχῆμα, τῶν δὲ λοιπῶν τριῶν φυσικῶς ἡπλωμένων καὶ μὴ χωριζομένων ἀπ' ἀλλήλων ἀλλὰ συνημμένων, τὸ τοιοῦτον ἐν μὲν τῇ εὐωνύμῳ δηλοῖ ι ἐν δὲ τῇ δεξιᾷ ρ. πάλιν τοῦ τετάρτου – τοῦ καὶ λιχανοῦ καλουμένου – ἐξηπλωμένου ἐπ' εὐθείας ὥσπερ ἴση γραμμῇ[60] τῶν δὲ λοιπῶν τριῶν συνημμένων καὶ πρὸς τὴν παλάμην ὡς ἐν σχήματι γωνίας ὑποκλινομένων μικρόν, τοῦ δὲ ἀντίχειρος ὑπεράνω τούτων κειμένου καὶ συνεγγύζοντος τῷ λιχανῷ, κ τὸ τοιοῦτον δηλοῖ καὶ ἐν τῇ δεξιᾷ σ. τοῦ λιχανοῦ πάλιν καὶ τοῦ ἀντίχειρος ἐκτεταμένως ὑποκλινομένων |$_{173v}$ καὶ κατὰ τὸ ἄκρον αὐτοῖς ἐγγιζόντων, τῶν δὲ λοιπῶν τριῶν ἐκτεταμένων καὶ συνημμένων ὄντων ὡς ἄγονται παρὰ τῆς φύσεως, λ τὸ τοιοῦτον δηλοῖ καὶ ἐν τῇ ἑτέρᾳ τ. πάλιν τῶν τεσσάρων ἐπ' εὐθείας ἐκτεταμένων καὶ τοῦ ἀντίχειρος ὑπὲρ τὸν λιχανὸν ὥσπερ γάμμα κειμένου καὶ πρὸς τὸ ἔξωθεν ἀποβλέποντος μέρος, ἐν τῇ λαιᾷ δηλοῖ μ καὶ ἐν τῇ δεξιᾷ υ. πάλιν ὡσαύτως τῶν τεσσάρων ἡπλωμένων κατ' εὐθεῖαν καὶ[61] κεκολλημένων τοῦ δ' ἀντίχειρος ὥσπερ γάμμα ἐπὶ τοῦ ἔσωθεν μέρους κειμένου ἐπὶ τῷ στήθει τοῦ λιχανοῦ, ν δηλοῖ καὶ ἐν τῇ ἑτέρᾳ φ. τούτων δὲ οὕτως ἐχόντων καὶ τοῦ λιχανοῦ κυκλικῶς τῷ ἀντίχειρι ἐπιφερομένου ἄχρις ἂν προσψαύσῃ τῷ μέσῳ κονδύλῳ τοῦ πρώτου καὶ δευτέρου ἄρθρου τὸ δ' ἄκρον τοῦ αὐτοῦ λιχανοῦ τῷ στήθει συμπέσῃ τοῦ ἀντίχειρος, ξ δηλοῖ καὶ χ. πάλιν ὁμοίως τῶν τριῶν ἡπλωμένων – ὡς καὶ πολλάκις εἰρήκαμεν – συνημμένως κειμένου καὶ τοῦ ἀντίχειρος τῷ λιχανῷ καὶ κατὰ τὸ ἀκρώνυχον τοῦ ἀντίχειρος ἑλικοειδῶς ἐπιφερομένου τοῦ λιχανοῦ, ο δηλοῖ καὶ ψ. πάλιν τῶν τριῶν ἱσταμένων ὡς ἐν |$_{174r}$ σχήματι γωνίας καὶ πρὸς τὴν παλάμην δῆθεν βλεπόντων, τοῦ δ' ἀντίχειρος ἐπάνω τοῦ μέσου καὶ τρίτου δακτύλου τῷ τρίτῳ κονδύλῳ τῷ πρὸς τῇ ῥίζῃ ὄντι τοῦ αὐτοῦ δακτύλου κειμένου καὶ πρὸς τὴν παλάμην ἡρμοσμένου, καὶ τοῦ λιχανοῦ ἐπάνω τοῦ ἀντίχειρος κειμένου ἐπὶ τῷ πρώτῳ ἄρθρῳ αὐτοῦ ἄχρις οὗ τὸ τούτου ἄκρῳ[62] ἐπὶ τῷ στήθει συμπέσῃ τοῦ ἀντίχειρος, ὀγδοήκοντα τὸ τοιοῦτον δηλοῖ καὶ ω. αὖθις τὴν χεῖρα παλαιστοῦ δίκην συστείλας, ὀρθίου ὄντος τοῦ ἀντίχειρος, καὶ τοὺς τρεῖς ἐκτείνας δακτύλους, τὸν δὲ λιχανὸν ἀφεὶς ὡς ἀπὸ τῆς συστολῆς τοῦ γρόνθου ἐγένετο, τὸ τοιοῦτον σχῆμα ἐν μὲν τῇ εὐωνύμῳ χειρὶ δηλοῖ ἐννενήκοντα ἐν δὲ τῇ δεξιᾷ ϡ.

Τὰ δὲ παρεπόμενα εἰσὶ ταῦτα ἓξ τὸν ἀριθμόν· πρῶτον ἔκθεσις τῶν στοιχείων, δεύτερον σύνθεσις[63], τρίτον ἀφαίρεσις, τέταρτον πολλαπλασιασμός, πέμπτον μερισμός, ἕκτον εὕρεσις τῆς τετραγωνικῆς πλευρᾶς. καὶ περὶ μὲν τῆς ἐκθέσεως τῶν στοιχείων εἴρηται· νυνὶ δὲ καὶ περὶ τῶν ἄλλων εἰρήσεται.

περὶ συνθέσεως. β

σύνθεσις μὲν οὖν ἐστὶν ἕνωσις δύο |$_{174v}$ καὶ τριῶν ἀριθμῶν εἰς ἑνὸς ποσότητα· οἷον ἓν καὶ δύο, τρία· γ καὶ γ, ς· ς καὶ δ, ι· ι καὶ ε, ιε· ιε καὶ ς, κα· κα καὶ ζ, κη· κη καὶ η, λς· λς καὶ θ, με· ἰδοὺ τὰ δύο μετὰ τῆς μονάδος συντεθέντα τὸν τρία ἀριθμὸν ἀπήρτισαν, καὶ πάλιν ὁ γ μετὰ τοῦ γ, ς, καὶ ἑξῆς.

περὶ ἐκβολῆς. γ

Ἐκβολὴ δέ ἐστιν ἀφαίρεσις ἥττονος ἀριθμοῦ ἀπὸ μείζονος· ἀεὶ γὰρ ὁ μέλλων ἐκβληθήσεσθαι ἐλάττων δεῖ εἶναι τοῦ ἀφ' οὗ ἐκβάλ{λ}εται. ἔστω δὲ καθ' ὑπόδειξιν ὅτι βούλομαι ἀφελεῖν ἀπὸ τῶν με, θ· καταλιμπάνεται δὴ λς· καὶ πάλιν ὀκτὼ ἀπὸ τῶν λς· καταλιμπάνεται κη· καὶ ζ ἀπὸ τοῦ κη· λοιπὰ ἔμειναν κα· καὶ ς τοῦ κα· λοιπὰ ιε· καὶ ἐπὶ τῶν ἄλλων ἡ αὐτὴ ἀκολουθία. δήλη δέ σοι γενήσεται ἥ τε ἐκβολὴ καὶ ἡ σύνθεσις ἀπὸ τῆς ἔμπροσθεν παρ' ἡμῶν ἐκτεθησομένης ταύλας, ὡς ἀπὸ τοῦ σοφωτάτου Παλαμήδους ἐμάθομεν, ἀλλὰ δὴ καὶ ὁ πολλαπλασιασμός.

περὶ πολλαπλασιασμοῦ. δ

60 ἴσῃ γραμμῇ corruptum **B** : ἴσῃ συγγραμμῇ **V**
61 καὶ s.l. suppl. m.2 **B**
62 ἄκρῳ **BV** : lege ἄκρον
63 δεύτερον σύνθεσις marg. suppl. m.2 **B**

Ἀριθμὸς ἀριθμὸν πολλαπλασιάζειν λέγομεν ὅταν, ὅσαι εἰσὶν ἐν αὐτῷ μονάδες, τοσαυτάκις |$_{175r}$ συντεθῇ ὁ πολλαπλασιαζόμενος καὶ γένηταί τις ἕτερος. οἷον ἐπὶ παραδείγματος, τετράκις τὰ τέσσαρα, ις· πεντάκις τὰ[64] η, μ. ἰστέον δὲ ὅτι, ὅταν ὁ αὐτὸς ἀριθμὸς ἑαυτὸν πολλαπλασιάσῃ, τότε ὁ γινόμενος ἀριθμὸς τετράγωνος ἐστὶν ἰσόπλευρος· ὅταν δὲ ἀριθμὸς τὸν μονάδι ἐλάττονα ἑαυτοῦ ἢ μείζονα πολλαπλασιάσῃ, τότε ὁ γινόμενος (τετράγωνος) ἐστὶν ἐπιμήκης· ὅταν δὲ ἀριθμὸς ἑαυτὸν πολλαπλασιάσῃ, εἶτα τὸν πολλαπλασιασθέντα πάλιν ὁ αὐτός, τότε ὁ γινόμενος κύβος ἐστί.

ε. περὶ μερισμοῦ

Μερισμὸς δέ ἐστιν, ὅταν μερίζοντες ἀριθμὸν πρὸς ἀριθμὸν σκοπῶμεν τί ἑκάστῃ μονάδι τοῦ παρ’ ὃν ὁ μερισμὸς γίνεται ἐπιβάλλει, οἷον ὅταν τὸν ιβ ἐπὶ τὸν γ μερίζοντες σκοπῶμεν τί ἑκάστῃ μονάδι[65] τοῦ [[δ]] γ ἐπιβάλλει· ἐπιβάλλουσι δὲ τέσσαρες μονάδες, ἐπεὶ καὶ τρὶς τὰ τέσσαρα, ιβ. μερίζεται δὲ καὶ ἐλάττων ἀριθμὸς πρὸς μείζονα· ἔνθα σκοπεῖται ἑκάστῃ μονάδι τοῦ μείζονος ἀριθμοῦ τί μέρος μονάδος ἐπιβάλλει. οἷον ὅταν τὸν δ ἐπὶ τὸν ις μερίζοντες σκοπῶμεν τί μέρος μονάδος |$_{175v}$ ἑκάστῃ τοῦ ις ἐπιβάλλει· ἐπιβάλλει δὲ τέταρτον, ἐπεὶ τετράκις τὰ τέσσαρα, ις – ὅσαι γὰρ μονάδες ἐπιβάλλουσιν ἑκάστῃ μονάδι τοῦ ἐλάττονος τοῦ μείζονος ἐπ’ αὐτὸν μεριζομένου, εἰς τοσαῦτα μέρη διαιρεῖν δεῖ τὴν μονάδα τοῦ ἐλάττονος ἐπὶ τὸν μείζονα μεριζομένου καὶ νομίζειν ἕκαστον μόριον ἑκάστῃ μονάδι ἐπιβάλλειν. καὶ τοσαῦτα μὲν περὶ μερισμοῦ ἔστωσαν.

περὶ τῆς (τετραγωνικῆς) πλευρᾶς

Πλευρὰ δὲ τοῦ μὲν ἀληθοῦς τετραγώνου δήλη σχεδὸν πᾶσιν· ὁ γὰρ πολλαπλασιασθεὶς ἐφ’ ἑαυτὸν ἀριθμὸς καὶ ἀποτελέσας τὸν (τετράγωνον) ἀριθμὸν οὗτος ἐστὶν ἡ πλευρὰ αὐτοῦ. τοῦ δὲ μὴ ἀληθοῦς τετραγώνου οὐ ῥᾳδία εἰς κατάληψιν καὶ διδάσκοντος αὐτὴν τινός· διὸ τὸν περὶ αὐτῆς λόγον ἐν ἄλλοις ἐταμιεύσαμεν.

ἄριστον δ’ ἂν εἴη καὶ περὶ τῆς τάξεως καὶ τῆς ἀναλογίας τῶν ἀριθμῶν διαλαβεῖν. εἰσὶ δὴ τῶν ἀριθμῶν τάξεις ἐννέα, ἐκ τῆς ὑπερκοσμίου καὶ νοερᾶς ἐννάδος τὴν μίμησιν ἔχουσαι, καὶ ὥσπερ ἐκεῖναι τὰς ἐλλάμψεις ἀπὸ τοῦ πρώτου καὶ ἀϊδίου φωτὸς ἔχουσιν, οὕτω κἀνταῦθα |$_{176r}$ οἱ ἀριθμοὶ ἐκ τῆς μονάδος τὴν γένεσιν ἔχοντες κατὰ τὴν τάξιν αὐτῶν ἔχουσι καὶ τὰς δυνάμεις, οἱ πρῶτοι πρώτως καὶ οἱ ὕστατοι ὕστατον· πάντες δ’ ὡς ἔφημεν ἀπὸ μονάδος τὴν γένεσιν ἔχουσιν· ἡ γὰρ μονὰς ἀριθμὸς οὐκ οὖσα γενητικὴ ἐστὶν ἀριθμῶν, πηγὴ οὖσα καὶ ῥίζα καὶ ἀφορμὴ πλήθους παντός, εἰκόνα σῴζουσα θείου· ἐρωτώμενοι γὰρ τί ἐστιν ἀριθμὸς φαμὲν σωρεία μονάδων ἢ μονάδων σύνθεσις.

περὶ τῆς τάξεως τῶν ἀριθμῶν

καὶ πρωτίστη μὲν τάξις πασῶν[66] ἀριθμῶν αἱ μονάδες πεφύκασι, δευτέρα δ’ αὖ αἱ δεκάδες, τρίτη αἱ ἑκατοντάδες, τετάρτη [[αἱ]][67] χιλιοντάδες, πέμπτη μοναδικαὶ μυριάδες, ἕκτη δεκαδικαὶ μυριάδες[68], ἑβδόμη ἑκατονταδικαὶ μυριάδες, ὀγδόη χιλιονταδικαὶ μυριάδες, καὶ ἐννάτη μυριονταδικαὶ μυριάδες[69], περαιτέρω δὲ τούτων τάξιν ἀριθμῶν οὐκ ἔστιν εὑρεῖν. πρόσχες δὴ ὅπως καὶ ἡ τούτων ἀναλογία προχωρεῖ[70]· ταῖς ἐννέα μονάσι μονάδα μίαν προσθεὶς δεκάδα μίαν ἐπλήρωσας· οὐκοῦν καὶ ταῖς ἐννέα δεκάσι |$_{176v}$ δεκάδα μίαν προσθεὶς ἑκατοντάδα ἀναλόγως τελέσεις, καὶ ἐν ταῖς λοιπαῖς τάξεσι τῶν ἀριθμῶν ἡ αὐτὴ ἐστὶν ἀκολουθία.

ὑπάρχουσι δὲ πάντων τῶν ἀριθμῶν αἱ μονάδες θεμέλιοι· πρὸς τὴν τάξιν γὰρ καὶ κλῆσιν ἑνὸς ἑκάστου τῶν ἀριθμῶν ἀπὸ μιᾶς ἑκάστης τάξεως τῶν μονάδων λαμβάνεις θεμέλιον.

περὶ τοῦ θεμελίου αὐτῶν

οἷον εἰ θέλεις εὑρεῖν τοῦ ι καὶ ρ ἐν ταῖς μονάσι θεμέλιον, λαμβάνεις τὴν μονάδα, ἥτις ἐστὶ πρώτη τῶν μετ’ αὐτὴν μονάδων, ὡς καὶ ὁ ι καὶ ρ πρῶτοι ἀριθμοὶ τῶν κατ’ αὐτοὺς εὑρίσκονται τάξεων. ὡσαύτως πάλιν καὶ τοῦ κ καὶ σ βάθρον ἐστὶν ἡ δυάς, καὶ τῶν λ καὶ τ ἐστὶν ἡ τριάς, τῶν μ καὶ υ ἡ

[64] τὰ s.l. m.1 **B**
[65] μονάδι s.l. m.1 **B**
[66] πασῶν sic **BV**
[67] αἱ eras. m.2 **B**
[68] ἕκτη δεκαδικαὶ μυριάδες marg. m.2 **B**
[69] ὀγδόη — μυριονταδικαὶ μυριάδες marg. m.2 **B**
[70] προχωρεῖ] προ– m.2 **B**

τετράς[71], τῶν δὲ ν καὶ φ ἐστιν ἡ πεντάς[72], τῶν δὲ ξ καὶ χ ἡ ἑξάς, τῶν ο καὶ ψ ἡ ἑπτάς, τῶν π καὶ ω ἡ ὀκτάς, τῶν ϙ καὶ ϡ ἡ ἐννάς, καὶ ἐπὶ τῶν ἑξῆς τάξεων ἡ αὐτὴ μέθοδος. ἵνα δὲ ἐπὶ ὑποδείγματος σαφέστερον γένηται τὸ λεγόμενον, |$_{177r}$ ἔστω ὅ τι ἠρωτήθης, λ καὶ ἐννενήκοντα πόσος ἀριθμὸς γίνεται, καὶ οὐ δύνῃ ῥᾳδίως ἐκ τῆς ἀμαθίας τοῦτον εὑρεῖν. λαβὼν δὴ ἀμφοτέρων τούτων ἀπὸ τῶν μονάδων τοὺς παρωνύμους καὶ ἰσοταγεῖς ἀριθμούς, ἀπὸ τοῦ σμικροῦ ἀριθμοῦ καὶ φανεροῦ τὸν μείζονα εὑρήσεις – τὸ γὰρ ἀφανὲς ἐκ τοῦ φανεροῦ, ὥσπερ ἄρα καὶ τὸ ἐναντίον ἐκ τοῦ ἐναντίου, ταχίστην ἔχει τὴν διάγνωσιν –· λαμβάνεται δὲ ἀντὶ μὲν τῶν λ ἡ τριάς, ἀντὶ δὲ τῶν ϙ ἡ ἐννάς (ἀναλογοῦσι γάρ), οἳ συντιθέμενοι δώδεκα ποιοῦσι μονάδας. οὐκοῦν καὶ αἱ τρεῖς δεκάδες μετὰ τῶν θ δεκάδων ἑνούμεναι δύο καὶ ι ποιοῦσι δεκάδας (ἤγουν ρ καὶ κ), ἐπειδή, ὡς προέφημεν, προστεθείσης τῇ ἐννάδι μονάδος μιᾶς δεκὰς ἀποτελεῖται μία, καὶ οὐκέτι ι μονάδας[73] ἔκτοτε τὸν γινόμενον ἀριθμὸν ὀνομάζομεν, ἀλλὰ δεκάδα μίαν ταύτην καλοῦμεν. ὡς ποιοῦντες ἑτέραν ἀρχὴν τῆς τῶν δεκάδων τάξεως καὶ πάλιν προστεθείσης ταῖς ἐννέα δεκάσι δεκάδος μιᾶς, οὐ[[κέτι]] |$_{177v}$ δέκα[74] δεκάδας τὸν γινόμενον ἀριθμὸν ὀνομάζομεν, ἀλλὰ μίαν ἑκατοντάδα, καὶ ἐπὶ τῶν ἄλλων ὁμοίως. λέγομεν οὖν τὰς ιβ μονάδας δεκάδα μίαν καὶ μίαν δυάδα (ἤγουν ι καὶ δύο), καὶ τὰς ιβ δεκάδας ὁμοίως ἑκατοντάδα μίαν καὶ μίαν εἰκάδα[75], ἤτοι ρκ. ἡ αὐτὴ δὲ καὶ ἐν τοῖς ἄλλοις ἀκολουθία, κἂν εἰς ἄπειρον δεήσειε προχωρεῖν.

λάμβανε δὲ καὶ ἑτέραν μέθοδον καθολικὴν εἰς παντὸς πολλαπλασιασμὸν ἀριθμοῦ τῶν προτέρων καινοπρεπεστέραν καὶ θαυμασιωτέραν καὶ ὥς πέρ τι τῶν ἄλλων εἰπεῖν ἐπισφράγισμα δι' ἐπιστημονικῶν καὶ φιλοσόφων κανονικῶν λόγων ἐκτεθειμένην, ἥτις καὶ ἔστιν αὕτη.

μέθοδος μονάδων

Αἱ μονάδες ἀλλήλαις συντιθέμεναι καὶ πολλαπλασιαζόμεναι γεννῶσι[76] μονάδας[77] καὶ[78] δεκάδας[79]. οἷον[80] ὡς ἐν ὑποδείγματι ἑξάκις τὰ ζ, μβ. ἰδοὺ τὰ μὲν ς καὶ ζ μονάδες εἰσὶ πάντως, καὶ πολλαπλασιασθεῖσαι ἔτεκον τὸν τεσσαρακοστόδυον ἀριθμόν, ὃς ἐστὶ τέσσαρες |$_{178r}$ δεκάδες καὶ δύο μονάδες.

Πάλιν αἱ μονάδες μετὰ τῶν δεκάδων πολλαπλασιαζόμεναι ποιοῦσι δεκάδας καὶ ἑκατοντάδας. καὶ ὅρα πῶς γίνεται. ἑξάκις τὰ ο, υκ. ἰδοὺ τὰ μὲν ἓξ εἰσὶ μονάδες τὰ δὲ ο δεκάδες, καὶ ἔτεκον τὰ υκ, ἃ εἰσὶ τέσσαρες ἑκατοντάδες καὶ δύο δεκάδες, καὶ ἡ ἀπόδειξις ἀληθής.

Αἱ δὲ μονάδες μετὰ τῶν ἑκατοντάδων ποιοῦσιν ἑκατοντάδας καὶ χιλιοντάδας. οἷον ἑξάκις τὰ ψ, ͵δσ. ἔχεις τὰ μὲν ἓξ μονάδας τὰ δὲ ψ ἑκατοντάδας, ἃ ἐγέννησαν τὸν ͵δσ. καὶ εἰσὶ πάντως τὰ μὲν ͵δ χιλιάδες δ τὰ δὲ σ δύο ἑκατοντάδες, καὶ ἀληθὴς ἡ ἀπόδειξις.

Καὶ αὖθις αἱ μονάδες ἐπὶ χιλιάδων μετρούμεναι γεννῶσι χιλιοντάδας καὶ μοναδικὰς μυριάδας· ἑξάκις γὰρ τὰ ͵ζ τετρακισμύρια δισχίλια γίνονται. ἰδοὺ τὰ μὲν ἕξ, ὡς πολλάκις εἰρήκαμεν, μονάδες εἰσὶ τὰ δὲ ͵ζ χιλιοντάδες, καὶ ἀπετέλεσαν|$_{178v}$ ἐκ τοῦ πολλαπλασίου τὰς δ μυριάδας καὶ τὰ δισχίλια. καὶ ὁμολογούμενον ὅτι τὰ δ΄ μοναδικαὶ μυριάδες εἰσὶ δ τὰ δὲ ͵β χιλιοντάδες β, καὶ δῆλον ὅ τὶ ἀληθής ἐστι πρὸς πάντα ἡ μέθοδος.

Μετὰ δὲ μυριάδων ἐπὶ μὲν τῶν μοναδικῶν μυριάδων ποιοῦσι δεκαδικὰς καὶ μοναδικὰς μυριάδας, ἐπὶ δὲ τῶν δεκαδικῶν, ἑκαντοκαδικὰς καὶ δεκαδικάς, καὶ ἐπὶ τῶν ἄλλων κατὰ τὴν ἀναλογίαν ἣν εἰρήκαμεν.

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71 τετράς] ε πεντάς fecit m.2 **B**

72 τῶν δὲ ν — πεντάς marg. m.2 **B**

73 ι μονάδας e corr. m.2 **B**

74 δέκα marg. m.1 **B**

75 εἰκάδα] ει– fecit e δε– m.2 **B**

76 ante γεννῶσι add. ποτὲ μὲν marg. m.2 **B**

77 post μονάδας add. *legi nequit* ὡς τὸ τριάκις τὰ γ, θ marg. m.2 **B**

78 καὶ eras. et scr. ποτὲ s.l. m.2 **B** : in textu ποτὲ δὲ **V**

79 post δεκάδας add. μόνον s.l. m.2 **B** : in textu μόνον dein spatium 12 litt. et in marg. *puto* ἅπαξ τὸ α, α *puto* δισσάκις τὰ β, δ, dein in textu τρισσάκις τὰ γ, θ **V**

80 post οἷον add. ὡς τὸ τετράκις τὰ ε, κ. καταχρηστικῶς ἐστὶ καὶ ὡς ἐπὶ τὸ πλεῖστον μονάδας καὶ δεκάδας. ὁ αὐτὸς δὲ λόγος καὶ εἰς τὰς λοιπὰς τάξεις τῶν ἀριθμῶν διαβήσεται ἀναλόγως marg. m.2 **B** : ante οἷον in textu ὡς τὸ τετράκις τὰ ε, κ. καταχρηστικῶς δὲ καθὼς ἐπὶ τὸ πλεῖστον μονάδας καὶ δεκάδας. ὁ αὐτὸς λόγος κτλ. **V**

μέθοδος δεκάδων[81]

Αἱ [[δὲ]] δεκάδες δ’ ἀλλήλαις καὶ αὗται πολλαπλασιαζόμεναι γεννῶσιν ἑκατοντάδας καὶ χιλιοντάδας. οἷον ἐπὶ παραδείγματος ἑξηκοντάκις τὰ π, ͵δω. ἰδοὺ πάντως τὰ μὲν ξ καὶ π δεκάδες εἰσὶ τὰ δὲ ͵δ εἴσι δ χιλιάδες καὶ τὰ ω ἑκατοντάδες η. μετὰ δὲ ἑκατοντάδων αἱ δεκάδες μετρούμεναι ποιοῦσι χιλιοντάδας καὶ μοναδικὰς μυριάδας. οἷον [[τὰ ἑβδομήκοντα]] ἑβδομηκοντάκις τὰ ϡ, ϛ΄͵γ. μετὰ χιλιοντάδων δὲ ποιοῦσι μυριάδας μοναδικὰς καὶ δεκαδικάς, ὡς τὸ π^{κις} ͵θ, ο΄β΄.

|$_{179r}$ μέθοδος ἑκατοντάδων

Αἱ ἑκατοντάδες ἀλλήλαις πολλαπλασιαζόμεναι ποιοῦσι μοναδικὰς μυριάδας καὶ δεκαδικάς. οἷον τετρακοσοντάκις[82] τὰ ἐννακόσια, λ΄ϛ΄. εἰσὶ γοῦν τὰ μὲν υ καὶ ϡ ἑκατοντάδες τὰ δὲ λ΄ϛ΄ δεκάδες καὶ μονάδες[83].

Μετὰ δὲ χιλιάδων ἀπαριθμούμεναι ποιοῦσι μυριάδας δεκαδικὰς καὶ ἑκατονταδικάς. οἷον ἑξακοσοντάκις τὰ ͵η, υπ μυριάδες.

μέθοδος χιλιοντάδων

Αἱ χιλιοντάδες καὶ αὗται ἀλλήλαις πολλαπλασιαζόμεναι ποιοῦσιν ἑκατονταδικὰς μυριάδας καὶ χιλιονταδικάς. ἔστω σοι καὶ τοῦτο καθ’ ὑπόδειξιν ἐπὶ παραδείγματος οὕτως. ἑπτάκις[84] χιλιοντάκις ͵ζ, τετρακισχίλιαι ἐννακόσιαι μυριάδες. ἰδοὺ τὰ μὲν ͵ζ καὶ ͵ζ χιλι{οντ}άδες[85] εἰσί, καὶ πολλαπλασιασθεῖσαι ἀπεγέννησαν τὰς ͵δ΄ϡ΄, καὶ δῆλον λοιπὸν ὡς πανταχοῦ ἀληθὴς εὑρέθη ἡ μέθοδος.

Εἰ δὲ μέχρις ἀπείρου τὸν πολλαπλασιασμὸν ἐπεκτείνῃς[86] καὶ μυριάδας ἐπὶ μυριάδας ἀριθμεῖν[87], τὰς προγραφείσας μεθόδους ὀφείλεις κρατεῖν καὶ ἀναλογίζεσθαι ὡς δεῖ· ἱκανὰ δὲ ταῦτα πάντως οἶμαι τοῖς εὖ φρονοῦσι ἔστωσαν πρὸς κατάληψιν. |$_{179v}$

πλινθὶς ἀρίστη τῆς τῶν ἀριθμητικῶν στοιχείων καταγραφῆς

θ	ϙ	ϡ	͵θ	θ΄	ϙ΄	ϡ΄	͵θ΄	θ΄΄
η	π	ω	͵η	η΄	π΄	ω΄	͵η΄	η΄΄
ζ	ο	ψ	͵ζ	ζ΄	ο΄	ψ΄	͵ζ΄	ζ΄΄
ϛ	ξ	χ	͵ϛ	ϛ΄	ξ΄	χ΄	͵ϛ΄	ϛ΄΄
ε	ν	φ	͵ε	ε΄	ν΄	φ΄	͵ε΄	ε΄΄
δ	μ	υ	͵δ	δ΄	μ΄	υ΄	͵δ΄	δ΄΄
γ	λ	τ	͵γ	γ΄	λ΄	τ΄	͵γ΄	γ΄΄
β	κ	σ	͵β	β΄	κ΄	σ΄	͵β΄	β΄΄
α	ι	ρ	͵α	α΄	ι΄	ρ΄	͵α΄	α΄΄
μονάδες	δεκάδες	ἑκατοντάδες	χιλιοντάδες	μοναδικαὶ μυριάδες	δεκαδικαὶ μυριάδες	ἑκαντοδικαὶ μυριάδες	χιλιονταδικαὶ μυριάδες	μυριοντάκις μυριάδες

81 καὶ ἔτι ἑκατοντάδας ὁμοῦ καὶ χιλιοντάδας add. marg. m.2 **B**

82 τετρακοσοντάκις **BV** : *puto* τετρακοσιοντάκις marg. **V**

83 δεκάδες καὶ μονάδες **BV** : lege δεκαδικαὶ μυριάδες καὶ μοναδικαί

84 ἑπτάκις s.l. m.1 **B**

85 –οντ– s.l. m.2 **B**

86 ἐπεκτείνῃς] ἐ fecit ex ἀ m.1 **B**

87 ἀριθμεῖς **V** : ἀριθμεῖν **B**

tabulae deperditae ante f. 180, a **V** suppletae

ἀρχὴ συνθέσεως καὶ τῆς ἀφαιρέσεως. δευτέρα στάσις.				β	θ	ια	β	ς	ς	ιβ	ς
				γ	γ	ς	γ	ς	ζ	ιγ	ς
				γ	δ	ζ	γ	ς	η	ιδ	ς
α	α	β	α	γ	ε	η	γ	ς	θ	ιε	ς
α	β	γ	α	γ	ς	θ	γ	ζ	ζ	ιδ	ζ
α	γ	δ	α	γ	ζ	ι	γ	ζ	η	ιε	ζ
α	δ	ε	α	γ	η	ια	γ	ζ	θ	ις	ζ
α	ε	ς	α	γ	θ	ιβ	γ	η	η	ις	η
α	ς	ζ	α	δ	δ	η	δ	η	θ	ιζ	η
α	ζ	η	α	δ	ε	θ	δ	θ	θ	ιη	θ
α	η	θ	α	δ	ς	ι	δ	ι	ι	κ	ι
α	θ	ι	α	δ	ζ	ια	δ	ι	κ	λ	ι
β	β	δ	β	δ	η	ιβ	δ	ι	λ	μ	ι
β	γ	ε	β	δ	θ	ιγ	δ	ι	μ	ν	ι
β	δ	ς	β	ε	ε	ι	ε	ι	ν	ξ	ι
β	ε	ζ	β	ε	ς	ια	ε	ι	ξ	ο	ι
β	ς	η	β	ε	ζ	ιβ	ε	ι	ο	π	ι
β	ζ	θ	β	ε	η	ιγ	ε	ι	π	ϙ	ι
β	η	ι	β	ε	θ	ιδ	ε	ι	ϙ	ρ	ι

κ	κ	μ	κ	μ	ξ	ρ	μ	ϙ	ϙ	ρπ	ϙ
κ	λ	ν	κ	μ	ο	ρι	μ	ρ	ρ	σ	ρ
κ	μ	ξ	κ	μ	π	ρκ	μ	ρ	σ	τ	ρ
κ	ν	ο	κ	μ	ϙ	ρλ	μ	ρ	τ	υ	ρ
κ	ξ	π	κ	ν	ν	ρ	ν	ρ	υ	φ	ρ
κ	ο	ϙ	κ	ν	ξ	ρι	ν	ρ	φ	χ	ρ
κ	π	ρ	κ	ν	ο	ρκ	ν	ρ	χ	ψ	ρ
κ	ϙ	ρι	κ	ν	π	ρλ	ν	ρ	ψ	ω	ρ
λ	λ	ξ	λ	ν	ϙ	ρμ	ν	ρ	ω	ϡ	ρ
λ	μ	ο	λ	ξ	ξ	ρκ	ξ	ρ	ϡ	͵α	ρ
λ	ν	π	λ	ξ	ο	ρλ	ξ	σ	σ	υ	σ
λ	ξ	ϙ	λ	ξ	π	ρμ	ξ	σ	τ	φ	σ
λ	ο	ρ	λ	ξ	ϙ	ρν	ξ	σ	υ	χ	σ
λ	π	ρι	λ	ο	ο	ρμ	ο	σ	φ	ψ	σ
λ	ϙ	ρκ	λ	ο	π	ρν	ο	σ	χ	ω	σ
μ	μ	π	μ	ο	ϙ	ρξ	ο	σ	ψ	ϡ	σ
μ	ν	ϙ	μ	π	π	ρξ	π	σ	ω	͵α	σ
				π	ϙ	ρο	π	σ	ϡ	͵αρ	σ

finis tabularum deperditarum

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τ	τ	χ	τ	χ	χ	͵ασ	χ	͵α	͵θ	α´	͵α
τ	υ	ψ	τ	χ	ψ	͵ατ	χ	͵β	͵β	͵δ	͵β
τ	φ	ω	τ	χ	ω	͵αυ	χ	͵β	͵γ	͵ε	͵β
τ	χ	ϡ	τ	χ	ϡ	͵αφ	χ	͵β	͵δ	͵ϛ	͵β
τ	ψ	͵α	τ	ψ	ψ	͵αυ	ψ	͵β	͵ε	͵ζ	͵β
τ	ω	͵αρ	τ	ψ	ω	͵αφ	ψ	͵β	͵ϛ	͵η	͵β
τ	ϡ	͵ασ	τ	ψ	ϡ	͵αχ	ψ	͵β	͵ζ	͵θ	͵β
υ	υ	ω	υ	ω	ω	͵αχ	ω	͵β	͵η	α´	͵β
υ	φ	ϡ	υ	ω	ϡ	͵αψ	ω	͵β	͵θ	α´͵α	͵β
υ	χ	͵α	υ	ϡ	ϡ	͵αω	ϡ	͵γ	͵γ	͵ϛ	͵γ
υ	ψ	͵αρ	υ	͵α	͵α	͵β	͵α	͵γ	͵δ	͵ζ	͵γ
υ	ω	͵ασ	υ	͵α	͵β	͵γ	͵α	͵γ	͵ε	͵η	͵γ
υ	ϡ	͵ατ	υ	͵α	͵γ	͵δ	͵α	͵γ	͵ϛ	͵θ	͵γ
φ	φ	͵α	φ	͵α	͵δ	͵ε	͵α	͵γ	͵ζ	α´	͵γ
φ	χ	͵αρ	φ	͵α	͵ε	͵ϛ	͵α	͵γ	͵η	α´͵α	͵γ
φ	ψ	͵ασ	φ	͵α	͵ϛ	͵ζ	͵α	͵γ	͵θ	α´͵β	͵γ
φ	ω	͵ατ	φ	͵α	͵ζ	͵η	͵α	͵δ	͵δ	͵η	͵δ
φ	ϡ	͵αυ	φ	͵α	͵η	͵θ	͵α	͵δ	͵ε	͵θ	͵δ
								͵δ	͵ϛ	α´	͵δ
								͵δ	͵ζ	α´͵α	͵δ

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͵δ	͵η	α´͵β	͵δ	τέλος συνθέσεως καὶ			α	ι	ι
͵δ	͵θ	α´͵γ	͵δ	τῆς ἀφαιρέσεως. ἀρχὴ			α	κ	κ
͵ε	͵ε	α´	͵ε	τοῦ πολλαπλασιασμοῦ.			α	λ	λ
͵ε	͵ϛ	α´͵α	͵ε	πολλαπλασιασμὸς ἁπλῶς			α	μ	μ
͵ε	͵ζ	α´͵β	͵ε	μοναδικῶν ἀριθμῶν μετὰ			α	ν	ν
͵ε	͵η	α´͵γ	͵ε	δεκαδικῶν μετὰ χιλιονταδικῶν.			α	ξ	ξ
͵ε	͵θ	α´͵δ	͵ε				α	ο	ο
͵ϛ	͵ϛ	α´͵β	͵ϛ	τρίτη στάσις. ἔναρξίς ἐστι			α	π	π
͵ϛ	͵ζ	α´͵γ	͵ϛ	μονάδων			α	ϙ	ϙ
͵ϛ	͵η	α´͵δ	͵ϛ				α	ρ	ρ
͵ϛ	͵θ	α´͵ε	͵ϛ	α	α	α	α	σ	σ
͵ζ	͵ζ	α´͵δ	͵ζ	α	β	β	α	τ	τ
͵ζ	͵η	α´͵ε	͵ζ	α	γ	γ	α	υ	υ
͵ζ	͵θ	α´͵ϛ	͵ζ	α	δ	δ	α	φ	φ
͵η	͵η	α´͵ϛ	͵η	α	ε	ε	α	χ	χ
͵η	͵θ	α´͵ζ	͵η	α	ϛ	ϛ	α	ψ	ψ
͵θ	͵θ	α´͵η	͵θ	α	ζ	ζ	α	ω	ω
α´	α´	β´	α´	α	η	η	α	ϡ	ϡ
				α	θ	θ	α	͵α	͵α
							α	͵β	͵β
							α	͵γ	͵γ
							α	͵δ	͵δ
							α	͵ε	͵ε

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α	͵ϛ	͵ϛ	β	π	ρξ	γ	γ	θ
α	͵ζ	͵ζ	β	ϙ	ρπ	γ	δ	ιβ
α	͵η	͵η	β	ρ	σ	γ	ε	ιε
α	͵θ	͵θ	β	σ	υ	γ	ϛ	ιη
α	α´	α´	β	τ	χ	γ	ζ	κα
β	β	δ	β	υ	ω	γ	η	κδ
β	γ	ϛ	β	φ	͵α	γ	θ	κζ
β	δ	η	β	χ	͵ασ	γ	ι	λ
β	ε	ι	β	ψ	͵αυ	γ	κ	ξ
β	ϛ	ιβ	β	ω	͵αχ	γ	λ	ϙ
β	ζ	ιδ	β	ϡ	͵αω	γ	μ	ρκ
β	η	ιϛ	β	͵α	͵β	γ	ν	ρν
β	θ	ιη	β	͵β	͵δ	γ	ξ	ρπ
β	ι	κ	β	͵γ	͵ϛ	γ	ο	σι
β	κ	μ	β	͵δ	͵η	γ	π	σμ
β	λ	ξ	β	͵ε	α´	γ	ϙ	σο
β	μ	π	β	͵ϛ	α´͵β	γ	ρ	τ
β	ν	ρ	β	͵ζ	α´͵δ	γ	σ	χ
β	ξ	ρκ	β	͵η	α´͵ϛ	γ	τ	ϡ
β	ο	ρμ	β	͵θ	α´͵η	γ	υ	͵ασ
			β	α´	β´	γ	φ	͵αφ
						γ	χ	͵αω

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γ	ψ	͵βρ	δ	λ	ρκ	δ	͵ζ	β´͵η
γ	ω	͵βυ	δ	μ	ρξ	δ	͵η	γ´͵β
γ	ϡ	͵βψ	δ	ν	σ	δ	͵θ	γ´͵ϛ
γ	͵α	͵γ	δ	ξ	σμ	δ	α´	δ´
γ	͵β	͵ϛ	δ	ο	σπ	ε	ε	κε
γ	͵γ	͵θ	δ	π	τκ	ε	ϛ	λ
γ	͵δ	α´͵β	δ	ϙ	τξ	ε	ζ	λε
γ	͵ε	α´͵ε	δ	ρ	υ	ε	η	μ
γ	͵ϛ	α´͵η	δ	σ	ω	ε	θ	με
γ	͵ζ	β´͵α	δ	τ	͵ασ	ε	ι	ν
γ	͵η	β´͵δ	δ	υ	͵αχ	ε	κ	ρ
γ	͵θ	β´͵ζ	δ	φ	͵β	ε	λ	ρν
γ	α´	γ´	δ	χ	͵βυ	ε	μ	σ[88]
δ	δ	ιϛ	δ	ψ	͵βω	ε	ν	σν
δ	ε	κ	δ	ω	͵γσ	ε	ξ	τ
δ	ϛ	κδ	δ	ϡ	͵γχ	ε	ο	τν
δ	ζ	κη	δ	͵α	͵δ	ε	π	υ
δ	η	λβ	δ	͵β	͵η	ε	ϙ	υν
δ	θ	λϛ	δ	͵γ	α´͵β	ε	ρ	φ
δ	ι	μ	δ	͵δ	α´͵ϛ	ε	σ	͵α
δ	κ	π	δ	͵ε	β´	ε	τ	͵αφ
			δ	͵ϛ	β´͵δ	ε	υ	͵β

88 {ε} λ ρν || {ε} μ σ marg. m.2 **B**

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ε	φ	͵βφ	ς	ι	ξ	ς	͵β	α´͵β
ε	χ	͵γ	ς	κ	ρκ	ς	͵γ	α´͵η
ε	ψ	͵γφ	ς	λ	ρπ	ς	͵δ	β´͵δ
ε	ω	͵δ	ς	μ	σμ	ς	͵ε	γ´
ε	ϡ	͵δφ	ς	ν	τ	ς	͵ς	γ´͵ς
ε	͵α	͵ε	ς	ξ	τξ	ς	͵ζ	δ´͵β
ε	͵β	α´	ς	ο	υκ	ς	͵η	δ´͵η
ε	͵γ	α´͵ε	ς	π	υπ	ς	͵θ	ε´͵δ
ε	͵δ	β´	ς	ϙ	φμ	ς	α´	ς´
ε	͵ε	β´͵ε	ς	ρ	χ	ζ	ζ	μθ
ε	͵ς	γ´	ς	σ	͵ασ	ζ	η	νς
ε	͵ζ	γ´͵ε	ς	τ	͵αω	ζ	θ	ξγ
ε	͵η	δ´	ς	υ	͵βυ	ζ	ι	ο
ε	͵θ	δ´͵ε	ς	φ	͵γ	ζ	κ	ρμ
ε	α´	ε´	ς	χ	͵γχ	ζ	λ	σι
ς	ς	λς	ς	ψ	͵δσ	ζ	μ	σπ
ς	ζ	μβ	ς	ω	͵δω	ζ	ν	τν
ς	η	μη	ς	ϡ	͵ευ	ζ	ξ	υκ
ς	θ	νδ	ς	͵α	͵ς	ζ	ο	υϙ

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ζ	π	φξ	η	η	ξδ	η	͵ε	δ´
ζ	ϙ	χλ	η	θ	οβ	η	͵ς	δ´͵μ[89]
ζ	ρ	ψ	η	ι	π	η	͵ζ	ε´͵ς
ζ	σ	͵αυ	η	κ	ρξ	η	͵η	ς´͵δ
ζ	τ	͵βρ	η	λ	ρμ[90]	η	͵θ	ζ´͵β
ζ	υ	͵βω	η	μ	τκ	η	α´	η´
ζ	φ	͵γφ	η	ν	υ	θ	θ	πα
ζ	χ	͵δσ	η	ξ	υπ	θ	ι	ϙ
ζ	ψ	͵δϡ	η	ο	φξ	θ	κ	ρπ
ζ	ω	͵εχ	η	π	χμ	θ	λ	σο
ζ	ϡ	͵ςτ	η	ϙ	ψκ	θ	μ	τξ
ζ	͵α	͵ζ	η	ρ	ω	θ	ν	υν
ζ	͵β	α´͵δ	η	σ	͵αχ	θ	ξ	φμ
ζ	͵γ	β´͵α	η	τ	͵βτ[91]	θ	ο	χλ
ζ	͵δ	β´͵η	η	υ	͵γσ	θ	π	ψκ
ζ	͵ε	γ´͵ε	η	φ	͵δ	θ	ϙ	ωι
ζ	͵ς	δ´͵β	η	χ	͵δω	θ	ρ	ϡ
ζ	͵ζ	δ´͵θ	η	ψ	͵εχ	θ	σ	͵αω
ζ	͵η	ε´͵ς	η	ω	͵ςυ	θ	τ	͵βψ
ζ	͵θ	ς´͵γ	η	ϡ	͵ζσ	θ	υ	͵γχ
ζ	α´	ζ´	η	͵α	͵η	θ	φ	͵δφ
			η	͵β	α´͵ς	θ	χ	͵ευ
			η	͵γ	β´͵δ	θ	ψ	͵ςτ
			η	͵δ	γ´͵β			

[89] δ´͵μ **B** : δ´͵η
[90] ρμ **B** : lege σμ
[91] ͵βτ **B** : lege ͵βυ sicut in **V**

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θ	ω	͵ζσ	ι	μ	υ	ι	͵ζ	ζ´
θ	ϡ	͵ηρ	ι	ν	φ	ι	͵η	η´
θ	͵α	͵θ	ι	ξ	χ	ι	͵θ	θ´
θ	͵β	α´͵η	ι	ο	ψ	ι	α´	ι´
θ	͵γ	β´͵ζ	ι	π	ω	κ	κ	υ
θ	͵δ	γ´͵ς	ι	ϙ	ϡ	κ	λ	χ
θ	͵ε	δ´͵ε	ι	ρ	͵α	κ	μ	ω
θ	͵ς	ε´͵δ	ι	σ	͵β	κ	ν	͵α
θ	͵ζ	ς´͵γ	ι	τ	͵γ	κ	ξ	͵ασ
θ	͵η	ζ´͵β	ι	υ	͵δ	κ	ο	͵αυ
θ	͵θ	η´͵α	ι	φ	͵ε	κ	π	͵αχ
πολλαπλασιασμὸς ἁπλῶς δεκαδικῶν ἀριθμῶν μετὰ δεκαδικῶν μετὰ ἑκατονταδικῶν καὶ μετὰ χιλιονταδικῶν. στάσις τετάρτη.			ι	χ	͵ς	κ	ϙ	͵αω
			ι	ψ	͵ζ	κ	ρ	͵β
			ι	ω	͵η	κ	σ	͵δ
			ι	ϡ	͵θ	κ	τ	͵ς
			ι	͵α	α´	κ	υ	͵η
			ι	͵β	β´	κ	φ	α´
			ι	͵γ	γ´	κ	χ	α´͵β
ι	ι	ρ	ι	͵δ	δ´	κ	ψ	α´͵δ
ι	κ	σ	ι	͵ε	ε´	κ	ω	α´͵ς
ι	λ	τ	ι	͵ς	ς´	κ	ϡ	α´͵η

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κ	͵α	β´	λ	χ	α´͵η	μ	σ	͵η
κ	͵β	δ´	λ	ψ	β´͵α	μ	τ	α´͵β
κ	͵γ	ς´	λ	ω	β´͵δ	μ	υ	α´͵ς
κ	͵δ	η´	λ	ϡ	β´͵ζ	μ	φ	β´
κ	͵ε	ι´	λ	͵α	γ´	μ	χ	β´͵δ
κ	͵ς	ιβ´	λ	͵β	ς´	μ	ψ	β´͵η
κ	͵ζ	ιδ´	λ	͵γ	θ´	μ	ω	γ´͵β
κ	͵η	ις´	λ	͵δ	ιβ´	μ	ϡ	γ´͵ς
κ	͵θ	ιη´	λ	͵ε	ιε´	μ	͵α	δ´
κ	α´	κ´	λ	͵ς	ιη´	μ	͵β	η´
λ	λ	ϡ	λ	͵ζ	κα´	μ	͵γ	ι´͵β
λ	μ	͵ασ	λ	͵η	κδ´	μ	͵δ	ι´͵ς
λ	ν	͵αφ	λ	͵θ	κζ´	μ	͵ε	κ´
λ	ξ	͵αω	λ	α´	λ´	μ	͵ς	κ´͵δ
λ	ο	͵βρ	μ	μ	͵αχ	μ	͵ζ	κ´͵η
λ	π	͵βυ	μ	ν	͵β	μ	͵η	λ´͵β
λ	ϙ	͵βψ	μ	ξ	͵βυ	μ	͵θ	λ´͵ς[92]
λ	ρ	͵γ	μ	ο	͵βω	μ	α´	μ´
λ	σ	͵ς	μ	π	͵γσ	ν	ν	͵βφ
λ	τ	͵θ	μ	ϙ	͵γχ	ν	ξ	͵γ
λ	υ	α´͵β	μ	ρ	͵δ	ν	ο	͵γφ
λ	φ	α´͵ε				ν	π	͵δ

[92] ι´͵β ι´͵ς κ´͵δ κ´͵η λ´͵β λ´͵ς **B** : lege ιβ´ ις´ κδ´ κη´ λβ´ λς´ sicut marg. ὁ βιβλιογράφος *corrig.* +ιβ´ *pro* ι´͵β || +ις´ *pro* ι´͵ς´ || +κδ´ *pro* κ´͵δ || +κη´ *pro* κ´͵η || +λβ´ *pro* λ´͵β´ || +λς´ *pro* λ´͵ς adnot. **V**

|184r

ν	ϙ	͵δφ	ξ	ξ	͵γχ	ξ	͵η	μ´η´
ν	ρ	͵ε	ξ	ο	͵δσ	ξ	͵θ	ν´δ´
ν	σ	α´	ξ	π	͵δω	ξ	α´	ξ´
ν	τ	α´͵ε	ξ	ϙ	͵ευ	ο	ο	͵δϡ
ν	υ	β´	ξ	ρ	͵ϛ	ο	π	͵εχ
ν	φ	β´͵ε	ξ	σ	α´͵β	ο	ϙ	͵ϛτ
ν	χ	γ´	ξ	τ	α´͵η	ο	ρ	͵ζ
ν	ψ	γ´͵ε	ξ	υ	β´͵δ	ο	σ	α´͵δ
ν	ω	δ´	ξ	φ	β´͵ε[93]	ο	τ	β´͵α
ν	ϡ	δ´͵ε	ξ	χ	γ´͵ϛ	ο	υ	β´͵η
ν	͵α	ε´	ξ	ψ	δ´͵β	ο	φ	γ´͵ε
ν	͵β	ι´	ξ	ω	δ´͵η	ο	χ	δ´͵β
ν	͵γ	ι´ε´	ξ	ϡ	ε´͵δ	ο	ψ	δ´͵θ
ν	͵δ	κ´	ξ	͵α	ϛ´	ο	ω	ε´͵ϛ
ν	͵ε	κ´ε´	ξ	͵β	ι´β´	ο	ϡ	ϛ´͵γ
ν	͵ϛ	λ´	ξ	͵γ	ι´η´	ο	͵α	ζ´
ν	͵ζ	λ´ε´	ξ	͵δ	κ´δ´	ο	͵β	ι´δ´
ν	͵η	μ´	ξ	͵ε	λ´	ο	͵γ	κ´α´
ν	͵θ	μ´ε´	ξ	͵ϛ	λ´ϛ´	ο	͵δ	κ´η´
ν	α´	ν´	ξ	͵ζ	μ´β´	ο	͵ε	λ´ε´
						ο	͵ϛ	μ´β´

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ο	͵ζ	μ´θ´	π	͵η	ξ´δ´	ϙ	α´	ϙ´
ο	͵η	ν´ϛ´	π	͵θ	ο´β´	πολλαπλασιασμὸς ἁπλῶς ἑκατονταδικῶν ἀριθμῶν μετὰ ἑκατονταδικῶν καὶ μετὰ χιλιονταδικῶν. στάσις πέμπτη.		
ο	͵θ	ξ´γ´	π	α´	π´			
ο	α´	ο´	ϙ	ϙ	͵ηρ			
π	π	͵ϛυ	ϙ	ρ	͵θ			
π	ϙ	͵ζσ	ϙ	σ	α´͵η			
π	ρ	͵η	ϙ	τ	β´͵ζ			
π	σ	α´͵ϛ	ϙ	υ	γ´͵ϛ			
π	τ	β´͵δ	ϙ	φ	δ´͵ε			
π	υ	γ´͵β	ϙ	χ	ε´͵δ			
π	φ	δ´	ϙ	ψ	ϛ´͵γ	ρ	ρ	α´
π	χ	δ´͵η	ϙ	ω	ζ´͵β	ρ	σ	β´
π	ψ	ε´͵ϛ	ϙ	ϡ	η´͵α	ρ	τ	γ´
π	ω	ϛ´͵δ	ϙ	͵α	θ´	ρ	υ	δ´
π	ϡ	ζ´͵β	ϙ	͵β	ι´η´	ρ	φ	ε´
π	͵α	η´	ϙ	͵γ	κ´ζ´	ρ	χ	ϛ´
π	͵β	ι´ϛ´	ϙ	͵δ	λ´ϛ´	ρ	ψ	ζ´
π	͵γ	κ´δ´	ϙ	͵ε	μ´ε´	ρ	ω	η´
π	͵δ	λ´β´	ϙ	͵ϛ	ν´δ´	ρ	ϡ	θ´
π	͵ε	μ´	ϙ	͵ζ	ξ´γ´	ρ	͵α	ι´
π	͵ϛ	μ´η´	ϙ	͵η	ο´β´	ρ	͵β	κ´
π	͵ζ	ν´ϛ´	ϙ	͵θ	π´α´	ρ	͵γ	λ´

[93] β´͵ε **BV** : lege γ´

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ρ	͵δ	μ´	σ	α´	σ´	υ	͵β	π´
ρ	͵ε	ν´	τ	τ	θ´	υ	͵γ	ρ´κ´
ρ	͵ς	ξ´	τ	υ	ι´β´	υ	͵δ	ρ´ξ´
ρ	͵ζ	ο´	τ	φ	ι´ε´	υ	͵ε	σ´
ρ	͵η	π´	τ	χ	ι´η´	υ	͵ς	σ´μ´
ρ	͵θ	ϙ´	τ	ψ	κ´α´	υ	͵ζ	σ´π´
ρ	σ´[94]	ρ´	τ	ω	κ´δ´	υ	͵η	τ´κ´
σ	σ	δ´	τ	ϡ	κ´ζ´	υ	͵θ	τ´ξ´
σ	τ	ς´	τ	͵α	λ´	υ	α´	υ´
σ	υ	η´	τ	͵β	ξ´	φ	φ	κ´ε´
σ	φ	ι´	τ	͵γ	ϙ´	φ	χ	λ´
σ	χ	ι´β´	τ	͵δ	ρ´κ´	φ	ψ	λ´ε´
σ	ψ	ι´δ´	τ	͵ε	ρ´ν´	φ	ω	μ´
σ	ω	ι´ς´	τ	͵ς	ρ´π´	φ	ϡ	μ´ε´
σ	ϡ	ι´η´	τ	͵ζ	σ´ι´	φ	͵α	ν´
σ	͵α	κ´	τ	͵η	σ´μ´	φ	͵β	ρ´
σ	͵β	μ´	τ	͵θ	σ´ο´	φ	͵γ	ρ´ν´
σ	͵γ	ξ´	τ	α´	τ´	φ	͵δ	σ´
σ	͵δ	π´	υ	υ	ι´ς´	φ	͵ε	σ´ν´
σ	͵ε	ρ´	υ	φ	κ´	φ	͵ς	τ´
σ	͵ς	ρ´κ´	υ	χ	κ´δ´	φ	͵ζ	τ´ν´
σ	͵ζ	ρ´μ´	υ	ψ	κ´η´	φ	͵η	υ´
σ	͵η	ρ´ξ´	υ	ω	λ´β´	φ	͵θ	υ´ν´
σ	͵θ	ρ´π´	υ	ϡ	λ´ς´	φ	α´	φ´
			υ	͵α	μ´			

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χ	χ	λ´ς´	ψ	͵ζ	υ´ϙ´	ϡ	͵ς	φ´μ´
χ	ψ	μ´β´	ψ	͵η	φ´ξ´	ϡ	͵ζ	χ´λ´
χ	ω	μ´η´	ψ	͵θ	χ´λ´	ϡ	͵η	ψ´κ´
χ	ϡ	ν´δ´	ψ	α´	ψ´	ϡ	͵θ	ω´[95]
χ	͵α	ξ´	ω	ω	ξ´δ´	ϡ	α´	ϡ´
χ	͵β	ρ´κ´	ω	ϡ	ο´β´	πολλαπλασιασμὸς ἁπλῶς χιλιονταδικῶν ἀριθμῶν. στάσις ἕκτη.		
χ	͵γ	ρ´π´	ω	͵α	π´			
χ	͵δ	σ´μ´	ω	͵β	ρ´ξ´			
χ	͵ε	τ´	ω	͵γ	σ´μ´			
χ	͵ς	τ´ξ´	ω	͵δ	τ´κ´			
χ	͵ζ	υ´κ´	ω	͵ε	υ´			
χ	͵η	υ´π´	ω	͵ς	υ´η´			
χ	͵θ	φ´μ´	[ω]	[͵ς]	φ´ς´			
χ	α´	χ´	ω	͵ζ	χ´δ´	͵α	͵α	ρ´
ψ	ψ	μ´θ´	ω	͵η	ψ´β´[96]	͵α	͵β	σ´
ψ	ω	ν´ς´	ω	͵θ		͵α	͵γ	τ´
ψ	ϡ	ξ´γ´	ω	α´	ω´	͵α	͵δ	υ´
ψ	͵α	ο´	ϡ	ϡ´	π´α´	͵α	͵ε	φ´
ψ	͵β	ρ´μ´	ϡ	͵α	ϙ´	͵α	͵ς	χ´
ψ	͵γ	σ´ι´	ϡ	͵β	ρ´π´	͵α	͵ζ	ψ´
ψ	͵δ	σ´π´	ϡ	͵γ	σ´ο´	͵α	͵η	ω´
ψ	͵ε	τ´ν´	ϡ	͵δ	τ´ξ´	͵α	͵θ	ϡ´
ψ	͵ς	υ´κ´	ϡ	͵ε	υ´ν´	͵α	α´	͵α´

[94] σ´ **B** : lege α´ sicut in **V**

[95] ω´ **B** : lege ω´ι´ sicut in **V**

[96] υ´η´ φ´ς´ χ´δ´ ψ´β´ **BV** : lege υ´π´ φ´ξ´ χ´μ´ ψ´κ´ sicut marg. *librarius* υ´π´ *pro* υ´η´ || φ´ξ´ *pro* φ´ς´ || χ´μ´ *pro* χ´δ´ || ψ´κ´ *pro* ψ´β´ adnot. **V**

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͵β	͵β	υ´	͵δ	͵ε	͵β´	͵ζ	͵ζ	͵δ´{θ´}
͵β	͵γ	χ´	͵δ	͵ς	͵β´υ´	͵ζ	͵η	͵ε´{ς´}
͵β	͵δ	ω´	͵δ	͵ζ	͵β´ω´	͵ζ	͵θ	͵ς´{γ´}
͵β	͵ε	͵α´	͵δ	͵η	͵γ´σ´	͵ζ	α´	͵ζ´
͵β	͵ς	͵α´σ´	͵δ	͵θ	͵γ´χ´	͵η	͵η	͵ς´υ´
͵β	͵ζ	͵α´υ´	͵δ	α´	͵δ´	͵η	͵θ	͵ζ´σ´
͵β	͵η	͵α´χ´	͵ε	͵ε	͵β´φ´	͵η	α´	͵η´
͵β	͵θ	͵α´ω´	͵ε	͵ς	͵γ´	͵θ	͵θ	͵η´ρ´
͵β	α´	͵β´	͵ε	͵ζ	͵γ´φ´	͵θ	α´	͵θ´
͵γ	͵γ	ϡ´	͵ε	͵η	͵δ´			
͵γ	͵δ	͵α´σ´	͵ε	͵θ	͵δ´φ´			
͵γ	͵ε	͵α´φ´	͵ε	α´	͵ε´			
͵γ	͵ς	͵α´ω´	͵ς	͵ς	͵γ´χ´	πολλαπλασιασμὸς ἁπλῶς μοναδικῶν μυριάδων ἐπὶ ὁμοίων		
͵γ	͵ζ	͵α´ρ´[97]	͵ς	͵ζ	͵δ´σ´			
͵γ	͵η	͵β´υ´	͵ς	͵η	͵δ´ψ´[98]			
͵γ	͵θ	͵β´ψ´	͵ς	͵θ	͵ε´υ´			
͵γ	α´	͵γ´	͵ς	α´	͵ς´	α´	α´	α´´
͵δ	͵δ	͵α´χ´[99]						

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α´	β´	β´´	οὕτως. ψαφισάτω τὸ α μετὰ τοῦ α, τὸ β μετὰ τοῦ β, εἶτα καὶ εἰς γ. τὸ α μετὰ τοῦ ἁπλοῦ α, μετὰ τοῦ β καὶ τὸ β[100] …	
α´	γ´	γ´´		
α´	δ´	δ´´		
α´	ε´	ε´´		
α´	ς´	ς´´		
α´	ζ´	ζ´´		
α´	η´	η´´		
α´	θ´	θ´´		
α´	ι´	ι´´		
τέλος τῶν ἀπὸ μονάδος μέχρι μυριάδων ἁπλῶν πολλαπλασιασμῶν				
ὁ δὲ διπλοῦς καὶ τριπλοῦς καὶ ἐπέκεινα πολλαπλασιασμὸς ἀεὶ γίνεται				

[97] ͵α´ρ´ **BV** : lege ͵β´ρ´ sicut marg. *librarius* ͵β´ρ´ *pro* ͵α´ρ´ adnot. **V**
[98] ͵δ´ψ´ **B** : lege ͵δ´ω´ sicut in **V**
[99] ͵α´χ´ **B** : ͵α´υ´ **V**
[100] ὁ δὲ διπλοῦς — τοῦ β καὶ το β **V** : legi nequit **B** : poasthac legi nequit **B** et non lexit **V**

A COMPARISON OF ANONYMUS B AND OF RHABDAS'S LETTER TO KHATZYKES

Let us first have a detailed look at the structure of *Anonymus* B. Titles, numbering of the sections, and rubricated letters induce the following partition[101]:

1) Overall title παράδοσις σύντομος καὶ σαφεστάτη τῆς ψηφηφορικῆς ἐπιστήμης ῥᾴστη τοῖς ἐθέλουσι ταύτην μετελθεῖν, ἥτις καὶ ἔχει οὕτως "concise and most clear exposition of the computational science, very easy for those who want to go through it, which also is as follows".
2) Short preface outlining the contents of the treatise.
3) Exposition of the numerical notation with Greek numerals. Title περὶ τῆς τῶν στοιχείων ἐκθέσεως "on the setting-out of digits". Letters representing thousands receive a lower left slanting stroke, myriads receive a superimposed trema (namely, a pair of dots); these two additional signs can be combined or iterated (that is, by superimposing tremas)[102]. Several examples. This section is assigned number 1 in the margin.
4) Very long exposition on how to represent numbers from 1 to 9,999 on the fingers of the hands. Title ἔκφρασις τοῦ δακτυλικοῦ μέτρου "description of numbering on fingers". Several examples.
5) Transitional sentences introducing the six παρεπόμενα "concomitant properties" of numbers[103]: these are the setting-out of digits (this is sect. 3) and the five basic operations, extraction of square root included.
6) Very short exposition περὶ συνθέσεως "on composition" (namely, addition). Mainly paradigmatic examples, cleverly conceived: the numbers from 1 to 9 are added in succession. This section is assigned number 2.
7) Short exposition περὶ ἐκβολῆς "on decurtation" (namely, subtraction). A definition and some paradigmatic examples, cleverly conceived insofar as they amount to a partial conversion of the sequence of additions in sect. 6; final reference to the addition and subtraction table (called

[101] An item numbered X in the list will be referred to as "sect. X".

[102] Recall that the Greek numerical notation (see M. N. Tod, The Alphabetic Numeral System in Attica. *ABSA* 45 [1950] 126–139) used in scientific texts is decimal but not positional, that number "zero" does not exist (but a sign for the "empty place" was used), and that the numbers from 1 to 999 are denoted by the 24 current alphabetical letters plus three additional ones, namely, letters ϛ (*digamma*, that in Byzantine manuscripts and in modern editions is always represented by the *sigma-tau* ligature known as *stigma*, almost identical to a form of *digamma* itself), ϙ (*koppa*), and ϡ (*sade* or *sampi*). These 27 letters are divided in three groups of nine, denoting in succession the units from 1 to 9, the decads from 10 to 90, and the hundreds from 100 to 900. Specific signs are added to the same digits to denote higher numerical orders, as *Anonymus* B explains: thus, ͵δχκθ is 4629. A slightly different notation can be found in the multiplication tables ascribed to Andronikos Doukas Sgouros (*PLP*, no. 25048) found in the manuscript Milano, Biblioteca Ambrosiana, E 80 sup. (*Diktyon* 42703), ff. 179r–195r (see also Vat. gr. 1058, ff. 41r–52v, without the ascription); they are affected by a systematic notational mistake (the author did not realize that a myriad of myriads coincides with ten-times-thousand myriads), run as far as $10^9 \times 10^9$, and are followed (ff. 195v–196r) by a tabular set-up of the names of the numbers associated with each numerical sign. In Sgouros' notation, myriads are denoted by a superimposed trema, myriads of myriads by a single superimposed dot. Further, an autograph scholium by Neophytos Prodromenos (*PLP*, no. 19254) in Par. gr. 1928 (*Diktyon* 51555), f. 15r, expounds a numerical notation with Hindu-Arabic numerals of the Eastern type: zero is not used; tens, hundreds, etc. are noted by superimposing a suitable number of small circles above the figures for units (P. Tannery, Le scholie du moine Néophytos sur les chiffres Hindous. *Revue archéologique*, 3e série, 5 [1885] 99–102, repr. Id., Mémoires Scientifiques IV. Toulouse–Paris 1920, 20–26). Of course, this amounts to not understanding that Hindu-Arabic numerals must be used positionally. Two short tabular texts on numerical notation adopting this convention are in Marc. gr. Z. 323, f. 487r–v; they are edited in J. L. Heiberg, Byzantinische Analekten. *Abhandlungen zur Geschichte der Mathematik* 9 (1899) 163–174: 172–174. Partitions of the 24 letters of the Greek alphabet according to their numerical values are in Par. suppl. gr. 920 (*Diktyon* 53604), f. 1r–v (10th c.).

[103] *Anonymus* 1252 (and hence also Planudes in his *Great Calculation*) and Rhabdas in his *Letter to Tzavoukhes* also refer to such a hexapartition: Allard, Premier 80.28–31; Allard, Planude 33.23–28; Tannery, Notice 118.15–23, respectively.

ταύλα)[104] that the anonymous says he himself (παρ' ἡμῶν) has set out before (ἔμπροσθεν). This section is assigned number 3.

8) Short exposition περὶ πολλαπλασιασμοῦ "on multiplication". A definition and some paradigmatic examples; definition of square, oblong (ἐπιμήκης), and cube numbers. This section is assigned number 4. We must stress that, in this and in the subsequent sections, no calculations are worked out: the "paradigmatic examples" simply amount to providing the operands and the result of the operation, and to checking that they fit the definition.
9) Short exposition περὶ μερισμοῦ "on division". A definition and some paradigmatic examples; division and "parts" of numbers. Dividing a greater number by a lesser one and vice versa; analogies between the two operations. This section is assigned number 5; it is the last section to which a number is assigned.
10) Very short exposition περὶ τῆς τετραγωνικῆς πλευρᾶς "on square root". The author simply declares that it is not an easy task to grasp it for non-square numbers, even with the help of a teacher, and for this reason he has treated the issue elsewhere.
11) Transition to the subsequent sections; no title. Arithmologically-oriented remarks on the monad and on the fact that there are only nine numerical orders.
12) Short exposition περὶ τῆς τάξεως τῶν ἀριθμῶν "on the ordering of numbers". Structure of the decimal system as far as the 8th numerical order, namely, the myriads of myriads.
13) Long introduction on the structure of the system of numerical orders; title περὶ τοῦ θεμελίου αὐτῶν "on the base of them". The "base" of a number (πυθμήν in the Neo-Pythagorean tradition) representing decads, hundreds, etc. is the monadic number corresponding to the multiplicity of decads, hundreds, etc.: thus, δ (4) is the base of μ (40). The introduction of the sign for zero and the resulting positional notation trivialize the whole affair: to us, the base of a number representing decads, hundreds, etc. can be immediately read from its representation as a numeral. A long, basic example of *addition* of non-monadic numbers using their bases. Final transition to the subsequent sections, which provide a methodical exposition of how to multiply a number of a given order by a number of another order.
14) Long exposition on multiplying monadic numbers by themselves and by decads, hundreds, thousands, and myriads. Title μέθοδος μονάδων "procedure for units". One example each. The two numbers multiplied in the examples always have the "bases" 6 and 7.
15) Very short exposition on multiplying decadic numbers by themselves and by hundreds and thousands. Title μέθοδος δεκάδων "procedure for decads". One example each.
16) Very short exposition on multiplying hundreds by themselves and by thousands. Title μέθοδος ἑκατοντάδων "procedure for hundreds". One example each.
17) Very short exposition on multiplying thousands by themselves. Title μέθοδος χιλιοντάδων "procedure for thousands". One example.
18) *Explicit*: one might go on indefinitely but what has been said will suffice for the astute reader.
19) Table setting out the digits of the numerical orders from monads as far as myriads of myriads, according to the description in sect. 3. Title πλινθὶς ἀρίστη τῆς τῶν ἀριθμητικῶν στοιχείων καταγραφῆς "best block-diagram of the arithmetical digits".
20) Tables of addition and subtraction[105] and of multiplication of numbers, the latter in order from units to myriads. The tables are in fact organized as one single table, partitioned in στάσεις "in-

[104] This loan word is not uncommon in Byzantine writings; in technical treatises, it scores for instance 16 occurrences in Chioniades' *Syntaxis Persica*.

[105] These two operations are set out in the same table, since it is enough to read the table "from opposite sides" in order to shift from addition to subtraction. Of course, some dispositions of numbers are better suited to represent a table of additions, others to represent a table of subtraction. We shall see that Rhabdas modifies this table.

stalments". The leaf containing the beginning of the series is missing. A final table possibly setting out multiples and submultiples of the numbers within the decads, endowed with a very short prefacing text, was probably contained in f. 186v (severely faded away) and in a missing leaf.

If we compare the text in Barb. gr. 4 with Rhabdas' *First Letter*, it is apparent that the latter presents some conspicuous additions, some minor additions, two radical rewritings, several systematic changes, and a modification in the layout of the table of addition[106]. We adopt such a lexicon to describe the variant readings between the two versions of the text for the sake of definiteness; of course, this does not beg the question as to what the original version of the text is. The case will be argued in more detail in the following remarks; we shall see in particular that Rhabdas deftly inserted some of his additions without perturbing the syntax of the host sentence. Here is a list of the main variants; we refer to our text above and to the pages of Tannery's edition of Rhabdas' *First Letter*.

- Addition to the title: 86.1–4, σχεδιασθεῖσα ἐν Βυζαντίδι τῇ Κωνσταντίνου, παρὰ Νικολάου Σμυρναίου Ἀρταβάσδου ἀριθμητικοῦ καὶ γεωμέτρου τοῦ Ῥαβδᾶ, αἰτήσει τοῦ πανσεβάστου ἐπὶ τῶν δεήσεων κυροῦ Γεωργίου τοῦ Χατζύκη. Thus, Rhabdas added his own name and the one of the addressee; thus, effectively appropriating the text.
- A long addition to the preface (sect. 2), amounting to its whole initial segment: 86.6–17 from τὴν δήλωσιν to καί σε καί. This mainly consists of a verbatim "quotation" of the very beginning of Diophantos' *Arithmetica*. The quotation can be found, always in liminal position but with a variation in the extent of the transcribed passage, in both of Rhabdas' *Letters*[107]. The original sentence in *Anonymus* B is modified as follows[108]: δεῖ τὸν βουλόμενον μετελθεῖν τὴν τῶν ἀριθμῶν ἐπιστήμην τοῦτον τὸν τρόπον προχωρῆσαι → [… οὕτω σε] δεῖ [τοῦ ἔργου πρότερον ἄρξασθαι καί σε καὶ] τὸν βουλόμενον μετελθεῖν τὴν τῶν ἀριθμῶν ἐπιστήμην ~~τοῦτον τὸν τρόπον προχωρῆσαι~~. The Diophantine "quotation" entirely precedes the sentence introduced by οὕτω.
- A short addition to sect. 3: 88.24–25, from ἃ καὶ to ἀριθμούς.
- A short addition to sect. 3: 90.2–5, from ἐντεῦθεν to ἄχρι τῶν ϡ.
- An enrichment at the end of sect. 3: 90.9–11. The original sentence is modified as follows: καὶ ἑτέρων ἔτι ἐπιτεθεισῶν στιγμῶν → [ἃς καὶ διπλᾶς, ἤτοι μυριοντάκις μυριονταδικάς, μυριάδας κατονομάζομεν], καὶ [ἑξῆς ὁμοίως κατὰ προσθήκην] ~~ἑτέρων ἔτι ἐπιτεθεισῶν~~ στιγμῶν, [τριπλᾶς καὶ τετραπλᾶς λέγοντες· καὶ] ἔτι ἑτέρας [τιθέντες].
- Addition of ἤτοι ἀφαιρέσεως "that is, removal" to the title of sect. 7[109]. The addition amounts to a categorial mistake since *Anonymus* B defines "decurtation" as a species of "removal".
- Elimination of παρ' ἡμῶν "by us" in the reference to the table in the final clause of sect. 7.
- A long addition at the end of the section on division (sect. 9): 100.1–10. The addition repeats what precedes, adding a further, trivial case: division can occur greater by less, less by greater, and equal by equal.

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[106] Since Tannery did not edit all tables accompanying Rhabdas' *First Letter*, we compare the tables of *Anonymus* B with those in Vat. gr. 1411, ff. 12r–13r. As seen in note 16, this manuscript is the best witness of Rhabdas' *First Letter*.

[107] Tannery, Notice 86.6–15 and 118.3–10, respectively, to be compared with P. Tannery (ed.), Diophanti Alexandrini opera omnia cum Graeciis commentariis. I–II. Lipsiae 1893–95, I 2.3–17 and 2.3–13, respectively (Rhabdas just modifies some of the seven words opening the *Arithmetica*). Thus, the "quotation" in the *Letter to Tzavoukhes* is included in the one in the *First Letter*. In his *Letter to Tzavoukhes*, Rhabdas also mentions Diophantos: Tannery, Notice 118.14–15.

[108] Here and elsewhere, the added text is within brackets, possibly replaced by three dots; the deleted text is struck out.

[109] Planudes adds the same word to the title of the corresponding section of *Anonymus* 1252: compare Allard, Premier 82.8, and Allard, Planude 39.21.

- A long addition constituting the bulk of the section on square root (sect. 10): 100.15–102.7 from κατὰ μὲν τὸ to λεπτομερέστερον. The way this addition is operated is particularly clever: the anonymous declares that extracting square roots of non-square numbers is not easy; for this reason, he has dealt with the issue elsewhere. Rhabdas splits the sentence and makes it divaricate by inserting in the middle the procedure for computing a first-order approximation of a square root—what he declares to be not easy is now a method for getting a higher order aproximation; for this reason, he has dealt with the issue elsewhere. The original sentence is modified as follows: ‹πλευρὰ› τοῦ δὲ μὴ ἀληθοῦς τετραγώνου οὐ ῥᾳδία εἰς κατάληψιν καὶ διδάσκοντος αὐτήν τινος· διὸ τὸν περὶ αὐτῆς λόγον ἐν ἄλλοις ἐταμιεύσαμεν → ‹πλευρὰ› τοῦ δὲ μὴ ἀληθοῦς τετραγώνου [κατὰ μὲν τὸ πάντη παχυμερέστερον γίνεται οὕτως … κατὰ δὲ τὸ ἄγαν λεπτομερέστερον] οὐ ῥᾳδία εἰς κατάληψιν καὶ διδάσκοντος αὐτήν τινος· διὸ τὸν περὶ αὐτῆς λόγον ἐν ἄλλοις ἐταμιεύσαμεν.
- At the end of sect. 12: after 104.2 ἀριθμῶν. A sequence divided between two clauses is eliminated, but the resulting sentence is perfectly formed: καὶ ἐν ταῖς λοιπαῖς τάξεσι τῶν ἀριθμῶν ~~ἡ αὐτὴ ἐστὶν ἀκολουθία. ὑπάρχουσι δὲ πάντων τῶν ἀριθμῶν αἱ μονάδες θεμέλιοι~~· πρὸς [γὰρ] τὴν τάξιν ~~γὰρ~~ καὶ κλῆσιν κτλ. Note also the γάρ shifted to a more canonical position.
- A radical reconceptualization of sect. 13. By means of a series of appropriate interventions, the example is transformed into a *multiplication* of numerical orders using their bases. Rhabdas' intervention trivializes the text: he removes an example of a kind not to be found elsewhere while providing one of a kind that will figure again in the subsequent exposition. The two versions of the end of sect. 13 are here set out in parallel; our text is at lines 131–145, Rhabdas' at 104.11–24. The syntagms that Rhabdas simply modifies while retaining their function in the clause are in italics; the sequences that are not found in the other version are underlined.

Anonymus B

ἵνα δὲ ἐπὶ ὑποδείγματος σαφέστερον γένηται τὸ λεγόμενον, ἔστω ὅ τι ἠρωτήθης, *λ καὶ ἐννενήκοντα* πόσος ἀριθμὸς γίνεται, καὶ οὐ δύνῃ ῥᾳδίως ἐκ τῆς ἀμαθίας τοῦτον εὑρεῖν.

λαβὼν δὴ ἀμφοτέρων τούτων ἀπὸ τῶν *μονάδων* τοὺς παρωνύμους καὶ ἰσοταγεῖς ἀριθμούς, ἀπὸ τοῦ σμικροῦ ἀριθμοῦ καὶ φανεροῦ τὸν μείζονα εὑρήσεις – τὸ γὰρ ἀφανὲς ἐκ τοῦ φανεροῦ, ὥσπερ ἄρα καὶ τὸ ἐναντίον ἐκ τοῦ ἐναντίου, ταχίστην ἔχει τὴν διάγνωσιν –· λαμβάνεται δὲ ἀντὶ μὲν τῶν λ ἡ τριάς, ἀντὶ δὲ τῶν ρ ἡ *ἐννάς* (ἀναλογοῦσι γάρ), οἳ συντιθέμενοι *δώδεκα* ποιοῦσι μονάδας·

οὐκοῦν καὶ αἱ *τρεῖς* δεκάδες μετὰ τῶν θ δεκάδων *ἑνούμεναι δύο* καὶ ι ποιοῦσι *δεκάδας (ἤγουν ρ καὶ κ)*, ἐπειδή, ὡς προέφαμεν, προστεθείσης ἐννάδι μονάδος μιᾶς δεκὰς ἀποτελεῖται μία, καὶ οὐκέτι ι μονάδας ἔκτοτε τὸν γινόμενον ἀριθμὸν ὀνομάζομεν, ἀλλὰ δεκάδα μίαν ταύτην καλοῦμεν, ὡς ποιοῦντες ἑτέραν ἀρχὴν τῆς τῶν δεκάδων τάξεως καὶ πάλιν προστεθείσης ταῖς ἐννέα δεκάσι δεκάδος μιᾶς οὐκέτι δέκα δεκάδας τὸν γινόμενον ἀριθμὸν ὀνομάζομεν, ἀλλὰ μίαν ἑκατοντάδα, καὶ ἐπὶ τῶν ἄλλων ὁμοίως. λέγομεν οὖν τὰς ιβ μονάδας δεκάδα μίαν καὶ μίαν δυάδα (ἤγουν ι καὶ δύο), καὶ τὰς ιβ δεκάδας ὁμοίως, ἑκατοντάδα μίαν καὶ μίαν εἰκάδα, ἤτοι ρκ. ἡ αὐτὴ δὲ καὶ ἐν τοῖς ἄλλοις ἀκολουθία, κἂν εἰς ἄπειρον δεήσειε προχωρεῖν.

λάμβανε *δὲ καὶ ἑτέραν μέθοδον καθολικὴν* εἰς παντὸς πολλαπλασιασμὸν ἀριθμοῦ κτλ.

Rhabdas

ἵνα δὲ ἐπὶ ὑποδείγματος σαφέστερον γένηται τὸ λεγόμενον, ἔστω ὅ τι ἠρωτήθης, *τριακοντάκις τὰ ρ* πόσος ἀριθμὸς γίνεται, καὶ οὐ δύνῃ πάντως ῥᾳδίως ἐκ τῆς ἀμαθίας τοῦτον εὑρεῖν.

λαβὼν δὴ ἐξ ἀμφοτέρων τούτων ἀπὸ τῶν *μοναδικῶν* τοὺς παρωνύμους καὶ ἰσοταγεῖς ἀριθμούς, ἀπὸ τοῦ σμικροῦ ἀριθμοῦ καὶ φανεροῦ τὸν μείζονα εὑρήσεις – τὸ γὰρ ἀφανὲς ἐκ τοῦ φανεροῦ, ὥσπερ ἄρα καὶ τὸ ἐναντίον ἐκ τοῦ ἐναντίου, ταχίστην ἔχει τὴν διάγνωσιν –· λαμβάνεται δὲ ἀντὶ μὲν τῶν λ ἡ τριάς, ἀντὶ δὲ τῶν ρ ἡ *ἐννεάς* (ἀναλογοῦσι γάρ), οἳ καὶ πολλαπλῶς συντιθέμενοι *β* ποιοῦσι δεκάδας καὶ μονάδας ζ, ἤγουν ζ καὶ κ·

οὐκοῦν καὶ αἱ γ δεκάδες μετὰ τῶν θ δεκάδων *μετρούμεναι κ* καὶ ζ ποιοῦσιν *ἑκατοντάδας, ἤτοι ͵βψ*, ἐπείπερ οἱ δεκαδικοὶ ἀριθμοὶ μετὰ τῶν δεκαδικῶν ἀριθμῶν πολλαπλασιαζόμενοι, ἑκατονταδικοὺς ποιοῦσι καὶ χιλιονταδικοὺς καὶ ἔτι ἐξ ἀμφοτέρων μικτοὺς ὡς ἐν τοῖς ἐφεξῆς δηλωθήσεται.

λάμβανε *τοίνυν πρὸς τὰς τοιαύτας ἐπερωτήσεις καὶ ἐπιλύσεις καθολικὴν μέθοδον* εἰς παντὸς πολλαπλασιασμὸν ἀριθμοῦ κτλ.

- A radical rewriting, with expansions and many more examples, of sects. 14–17[110]. Such a rewriting is also motivated by the fact that the denominations of the numerical orders, which Rhabdas modifies (see just below), occur very frequently in these sections.
- Systematic modifications can be found in the titles of the sections and in the inscriptions of the tables (most notably by eliminating the numbered references to the "instalments").
- Systematic lexical modifications include disposing of the word ἀκολουθία "consequence, chaining" and, first and foremost, modifying all denominations of numerical orders from μονάδες, δεκάδες, ἑκατοντάδες, "units", "decads", "hundreds", etc. into μοναδικοὶ ἀριθμοί, δεκαδικοὶ ἀριθμοί, ἑκατονταδικοὶ ἀριθμοί, "unitary numbers", "decadic numbers", "centenary numbers", etc.
- Specific lexical changes, all amounting to corrections or to *lectiones faciliores*: εὐωνύμῳ (line 27 of our edition) → λαιᾷ (Tannery's page 90.15); ἑτέρᾳ (38) → δεξιᾷ (92.7); δηλοῖ ι (51) → χειρὶ σημαίνει δέκα (92.26); ἴσῃ γραμμῇ (52) → Ι γράμμα (94.2)[111]; ἱσταμένων (65) → συνημμένως ὑποκλινομένων (94.23); ἄκρῳ (69) → ἄκρον (96.2)[112]; ἔμειναν (84) → οὖν (96.23); ἀκολουθία (84) → μέθοδος (96.24); λέγομεν (88) → λέγεται (98.2); (τετράγωνος) ἐστὶν ἐπιμήκης (92) → ἀριθμὸς ἑτερομήκης λέγεται (98.8)[113]; ἐπεί (97) → ἐπειδή (98.16); πρώτως (112) → πρότερον (102.16); ἐπλήρωσας (121) → ἐποίησας (102.27); insertion of three coordinants δέ (cf. lines 130–131 and page 104.9–10); insertion of πάντως and ἐξ (cf. line 133 and page 104.12–13).
- The table in sect. 19 is eliminated.
- The layout of the addition and subtraction table in sect. 20 is radically modified. To understand how this is done, recall that an addition or subtraction table such as those set out in our treatises only features values of monadic numbers, decads, hundreds, etc., as operands without mixing them: we thus never find 327 (τκζ) added to 12 (ιβ), or 100 (ρ) added to 50 (ν), but 10 (ι) added to 90 (ϙ), etc. In *Anonymus* B, a self-contained tabular unit of the addition and subtraction table is so framed: first column, a sequence of equal numerical values; second column *from top to bottom*, all numerical values starting from the one set out in the first column until the last number before the next numerical order is reached; third column, their sum; fourth column (in red), identical to the first column. In this way, the self-contained tabular units of such a table progressively reduce their length from nine items to one. Rhabdas modifies each self-contained unit in the following way: first column, a sequence of nine equal numerical values; second column *from bottom to top*, all nine consecutive numerical values following the one set out in the first column, even if they are numbers mixing two numerical orders; third column, their difference; fourth column, identical to the first column, but each time the number of items set out is reduced by one[114]. In this way, the self-contained tabular units of such a table are all of equal length. Thus, if read left-right, the table of *Anonymus* B is an addition table, Rhabdas' is

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110 Rhabdas' text also seems to retain traces of the reviser's correction at f. 177v.

111 The reading in *Anonymus* B is hardly meaningful and almost certainly a mistake.

112 The reading in *Anonymus* B is ungrammatical and certainly a mistake.

113 No occurrences of ἐπιμήκης in this sense are found in the Greek mathematical corpus; note that the adjective qualifies the word "square". It is standard Neo-Pythagorean doctrine that a ἑτερομήκης is a number of the form $n(n+1)$: Theon of Smyrna, *Exp*., 26.21–22 ed. E. Hiller, Theonis Smyrnaei philosophi platonici expositio rerum mathematicarum ad legendum Platonem utilium. Lipsiae 1878; Nicomachos, *Ar*. II.17.1 and II.18.2; Iamblichos, *in Nic*., 74.19–23 ed. E. Pistelli, Iamblichi in Nicomachi arithmeticam introductionem liber. Lipsiae 1894 (= 4.76, 142.33–35 ed. N. Vinel, Jamblique, In Nicomachi Arithmeticam [*Mathematica Graeca Antiqua* 3]. Pisa–Roma 2014). This is the designation retained in Chis. R.IV.20 (see note 16 above).

114 Thus, we find nine α, eight β, seven γ, and so on.

a subtraction table, with the minuend placed in the second column. Let us compare the addition and subtraction table for 300 (τ) as an example:

Anonymus B

τ	τ	χ	τ
τ	υ	ψ	τ
τ	φ	ω	τ
τ	χ	ϡ	τ
τ	ψ	͵α	τ
τ	ω	͵αρ	τ
τ	ϡ	͵ασ	τ

Rhabdas

τ	͵ασ	ϡ	τ
τ	͵αρ	ω	τ
τ	͵α	ψ	τ
τ	ϡ	χ	τ
τ	ω	φ	τ
τ	ψ	υ	τ
τ	χ	τ	τ
τ	φ	σ	
τ	υ	ρ	

- Elimination of the last nine entries of the table of multiplication; these are the ones in the table at f. 186v of Barb. gr. 4.
- Santamaura's transcription confirms that on f. 186v of Barb. gr. 4, before the table of multiples and of partition that very likely completes *Anonymus* B, no reference is present to more complex expositions of multiplication and division to be found in the Ἰνδικὴ μεγάλη ψηφοφορία "Great Indian Calculation", as we instead read in Rhabdas' *First Letter*[115].

The character of the variant readings listed above makes it certain that Rhabdas' *First Letter* is a revision of the anonymous text, and not, as Tannery submitted on the basis of the very limited evidence he had at his disposal, the latter a debased recension of the former. On these same grounds, we may safely exclude the possibility that what we read in Barb. gr. 4 is Rhabdas' first, youthful redaction of his own treatise. The issue seems to us to be settled by the idle addition to the section on division, by the clever syntactical divarication of a sentence in order to accommodate a whole procedure of extraction of a square root, by the reconceptualization of sect. 13, a useless move since the final result anticipates the subsequent sections. Apart from two substantial rewritings and a systematic lexical change, Rhabdas' recension displays the same character as almost all Byzantine recensions of previous (and possibly Byzantine) mathematical texts: they tend to expand[116] and to trivialize the original.

THE AUTHOR OF ANONYMUS B
THE FLOURISHING OF LOGISTIC TREATISES IN THE MIDDLE OF 13TH CENTURY

The same tendency to expand is displayed by a much-celebrated recension of a Byzantine technical text, namely, the one in which Hindu-Arabic numerals were introduced in Byzantium. The original text, namely, *Anonymus* 1252, is a handbook of logistic using a positional decimal system with "Indian" figures; its title is Ψηφηφορία κατ' Ἰνδοὺς ἡ λεγομένη μεγάλη *Great Calculation According to the Indians*[117]. It includes a description of the decimal notation used; methods for addition, subtrac-

[115] At Tannery, Notice 114.3 and 114.14.

[116] The shortening of the stretch of text of sect. 13 discussed above is of course induced by the fact that Rhabdas changes the type of example. One might wonder why the trend is that of expanding and not of compressing—the same phenomenon can also be perceived in Byzantine recensions of ancient Greek mathematical works, but there are structural reasons that explain this case: F. Acerbi, Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey. *Estudios bizantinos* 4 (2016) 133–213: 137–143. Maybe the rhetorical education encourages this attitude, or maybe it is just a scholarly habit: appropriation as production of a sort of compound "main text + commentary".

[117] Edition Allard, Premier. Allard's assessment of the manuscript tradition must certainly be reconsidered, as he blindly (and therefore fallaciously) availed himself of Mogenet's method for treating variant readings. The most ancient witnesses of

tion, multiplication, and division; an Easter Computus, assuming 1252 as the current year[118]; an exposition of calculations involving signs, degrees, and minutes on the zodiacal circle[119]; extraction of an approximate square root (first-order Heronian approximation). All methods are counter-checked (the check is called δοκιμή); for instance, the square root is checked by multiplication. Some cumbersomeness in the exposition comes from the proliferation of cases, supposedly required by the varying quantity of digits in a number and by the presence of zero (τζίφρα) among them. Western Arabic numerals are used[120]. The very beginning of the treatise suggests that it was part of a larger work[121].

Maximos Planudes' Ψηφηφορία κατ' Ἰνδοὺς ἡ λεγομένη μεγάλη *Great Calculation according to the Indians* is largely inspired by *Anonymus* 1252[122]. The structure is the same[123]; however, the Easter Computus is eliminated, the section on square root reaches to the second-order Heronian approximation and is enriched by a standard method, carried out within the sexagesimal system, based on Euclid, *Elements* II.4[124]. A final section with disparate problems is certainly spurious. Verbosity, a few alternative methods, and a more abundant set of examples make Planudes' treatise much longer than its source.

Thus, the following overall picture begins to take shape. In the second half of the 13th century, two parallel systematizations of logistic were redacted displaying altogether different goals: the one provides the basics of the decimal system in Greek notation, the other explains how to carry out computations resorting to Hindu-Arabic numerals and positional notation[125]. The latter was certainly

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Anonymus 1252 are Vat. gr. 184 (*Diktyon* 66815) (ca. 1270), ff. 2r–8r; Par. suppl. gr. 387 (*Diktyon* 53135), ff. 163r–180v (ca. 1306); Marc. gr. Z. 303 (*Diktyon* 69774), ff. 222v–228r (mid-14th century); Par. gr. 2988 (*Diktyon* 52630) (14th century), ff. 324r–341v. For Vat. gr. 184, see Tihon, Eudaimonoioannes; for Par. suppl. gr. 387, see Heronis Alexandrini opera quae supersunt omnia, IV iv–vii; M.-L. Concasty, Un manuscrit scolaire (?) de mathématiques. *Scriptorium* 21 (1967) 284–288; F. Acerbi – B. Vitrac (eds.), Héron d'Alexandrie, Metrica (*Mathematica Graeca Antiqua* 4). Pisa–Roma 2014, 437–439.

118 The date is given directly in the current era, not in the *anno mundi* era.

119 This is a mixed and periodic duodecimal-sexagesimal system in which 30 degrees = 1 sign; it was in use among 14th-century astronomers.

120 On Hindu-Arabic numerals in Byzantine scientific texts and manuscripts (where they are traditionally considered to appear for the first time during the 12th century) see K. Vogel, Buchstabenrechnung und indische Ziffern in Byzanz, in: Akten des XI. Internationalen Byzantinisten-Kongresses 1958. Munich 1960, 660–664, repr. Id., Kleinere Schriften zur Geschichte der Mathematik (*Boethius* 20). Stuttgart 1988, 452–456; N. Wilson, Miscellanea Palaeographica. *GRBS* 22 (1981) 395–404: 400–404; Ch. Burnett, Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the "Eastern Forms", in: From China to Paris: 2000 Years Transmission of Mathematical Ideas, ed. Y. Dold-Samplonius – J. W. Dauben – M. Folkerts – B. van Dalen (*Boethius* 46). Stuttgart 2000, 237–288.

121 *Anonymus* 1252 has no preface, and the first sentence is εἴπωμεν δὲ καὶ περὶ τῶν ψήφων τῆς ἀστρονομίας "and let us also speak about the calculations in astronomy" (Allard, Premier 80.2), to be compared with the almost identical liminal sentences ibid., 87.19 and 98.24, marking the beginning of the sections on division and on the sexagesimal system, respectively, and ibid., 101.27, marking the beginning of the section on multiplication within the sexagesimal system.

122 Edition Allard, Planude. The relation is obvious once one compares the two texts. We also know, from a letter by Planudes to George Bekkos (see n. 146 for the text), that he happened to own a book on κατ' Ἰνδοὺς ἀριθμός "number according to Indians" and that he was composing his own, in which he would have added expositions of the parts about how to find the square number nearest to a given non-square number, and the side of a given square number (*sic*). Planudes' treatise is partly extant in the autograph Ambros. Suppl. 157 sup. (*Diktyon* 43243): A. Turyn, Dated Greek Manuscripts of the Thirteenth and Fourteenth Centuries in the Libraries of Italy. I–II. Urbana–Chicago–London 1972: 78–81 and pl. 57; A. Allard, L'Ambrosianus Et 157 Sup., un manuscrit autographe de Maxime Planude. *Scriptorium* 33 (1979) 219–234.

123 The numerals are those employed by Persian astronomers, and also found in annotations in Vat. gr. 211 (*Diktyon* 66842) (beginning 14th century) and 1058 (middle 14th century).

124 This method is well-known at least since Theon, *Commentary on the Almagest*, 469.16–473.8 ed. A. Rome, Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste (*Studi e Testi* 54, 72, 106). I–III. Città del Vaticano 1931–43. Planudes' example is the same as Theon's.

125 The exposition of calculations involving signs, degrees, and minutes on the zodiacal circle in *Anonymus* 1252 explains how to carry out additions and subtractions, multiplications and very simple divisions within the sexagesimal system and using Hindu-Arabic notation. The first, short but complete, Byzantine computational primer within the sexagesimal system and using Greek notation is contained in §§ 1–6 and 26 of the astronomical section of Georges Pachymeres' *Quadrivium* (Pa-

redacted in the Nicaean period and contains sections that do not find parallels in the former (the Easter Computus and the exposition of calculations involving signs, degrees, and minutes on the zodiacal circle). Reading both treatises, the clear impression is that they do not come from the same author. Their lexicon displays remarkable differences[126]; the style of *Anonymus* 1252 is much less relaxed; it provides no definitions of the operations; its exposition is strictly finalized to explain computation techniques within the new notational framework by means of sometimes quite complex examples[127]. *Anonymus* B, on the other hand, is a primer to the decimal system in Greek numerical notation and to the *meaning* of the elementary arithmetical operations: as said, no such operation is explicitly carried out. The difference in style and goals between the two treatises can easily be perceived by comparing the entire section on multiplication in *Anonymus* B and the beginning of the same section in *Anonymus* 1252[128]:

Anonymus B

Ἀριθμὸς ἀριθμὸν πολλαπλασιάζειν λέγομεν ὅταν, ὅσαι εἰσὶν ἐν αὐτῷ μονάδες, τοσαυτάκις συντεθῇ ὁ πολλαπλασιαζόμενος καὶ γένηταί τις ἕτερος. οἷον ἐπὶ παραδείγματος, τετράκις τὰ τέσσαρα, ις· πεντάκις τὰ η, μ. ἰστέον δὲ ὅτι, ὅταν ὁ αὐτὸς ἀριθμὸς ἑαυτὸν πολλαπλασιάσῃ, τότε ὁ γινόμενος ἀριθμὸς τετράγωνος ἐστὶν ἰσόπλευρος· ὅταν δὲ ἀριθμὸς τὸν μονάδι ἐλάττονα ἑαυτοῦ ἢ μείζονα πολλαπλασιάσῃ, τότε ὁ γινόμενος (τετράγωνος) ἐστὶν ἐπιμήκης· ὅταν δὲ ἀριθμὸς ἑαυτὸν πολλαπλασιάσῃ, εἶτα τὸν πολλαπλασιασθέντα πάλιν ὁ αὐτός, τότε ὁ γινόμενος κύβος ἐστί.

Anonymus 1252

Ἰτέον δὲ καὶ τὴν τοῦ πολλαπλασιασμοῦ μέθοδον παραστῆσαι ὡς μεμαθήκαμεν. ἴσθι ὡς πολλαπλασιάζων ὀφείλεις γινώσκειν τὰς τρεῖς ταύτας μεθόδους, ἵνα εὐστόχως ποιῇς τὸν πολλαπλασιασμόν. καὶ μία μέν ἐστιν αὕτη. ἐὰν ὦσι ὅροι δύο μὲν ἄνωθεν καὶ δύο κάτωθεν καὶ ἔχωσι κατὰ τοὺς δευτέρους τόπους τὸ αὐτὸ σχῆμα, σύνθες τὸ εἰς τὸν πρῶτον τόπον καὶ τὸ εἰς τὸν δεύτερον τόπον μετὰ τοῦ ὄντος εἰς τὸν ἕτερον πρῶτον τόπον, εἶτα πολλαπλασίασον τοὺς τρεῖς ὁμαλοὺς συντεθέντας μετὰ τοῦ λοιποῦ τῶν ἐν τῷ δευτέρῳ τόπῳ, εἶτα τοὺς δύο τοὺς ἐν τοῖς πρώτοις τόποις πολλαπλασίασον πρὸς ἀλλήλους, καὶ ἑνώσας τὸν ἐκ τούτων πολλαπλασιασμὸν μετὰ τοῦ πρώτου πολλαπλασιασμοῦ ἕξεις τὸ ζητούμενον.

Anonymus B also introduces, in sect. 11, arithmological overtones that the author of *Anonymus* 1252 does not avail of[129]. To complete the picture, recall that the author of *Anonymus* B claims that he will deal elsewhere with procedures of approximation of a square root, and that, at the end of Rhabdas' *First Letter* but *not* at the end of *Anonymus* B, the reader is referred to more complete expositions of multiplication and division to be found in the "Great Indian Calculation". Thus the only explicit link between *Anonymus* B and *Anonymus* 1252 is severed: Rhabdas himself, who knew of Planudes' *Great Calculation*, added this reference.

One might wonder whether *Anonymus* 1252, and possibly also *Anonymus* B, was produced in Nicaea or in Latin Constantinople, and for what purposes. There are arguments supporting both hypotheses about its origin. Latin Constantinople was obviously the ideal place where interactions with Western mathematics could take place[130]. On the other hand, the oft-repeated claim that Planudes'

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chymeres, *PLP*, no. 22186, was a contemporary of Planudes). Byzantine scholars had easy access to such primers redacted in Late Antiquity, namely, a long dedicated section of the anonymous *Prolegomena to the Almagest* (see note 4 above) and a series of sparse examples within Theon's *Commentary on the Almagest*.

126 To give one example, *Anonymus* 1252 never uses the term ἀκολουθία, but it shares with *Anonymus B* the term ἐκβολή for "subtraction", glossed as ἀφαίρεσις both by Rhabdas and by Planudes.

127 Contrary to *Anonymus* B, *Anonymus* 1252 does not provide general definitions of the operations.

128 The text of *Anonymus* 1252 is at Allard, Premier 83.15–23 (we have modified the punctuation); the entire section goes as far as ibid., 85.5 (45 lines in all), and includes two worked-out examples and a detailed general explanation of the procedure to be used when more than two numbers are multiplied.

129 Proximity searches in the TLG database do not suggest any specific source for this long transitional argument. The combination ὑπερκόσμιος + νοερός is particularly frequent in Proclus and Damascius; the definition of number as σωρεία μονάδων "heap of units" is found in a number of commentators and lexicographers.

130 Recall that Leonardo Pisano (Fibonacci) first redacted his *Liber abbaci* in 1202, and that he revised it in 1228; we only read the revision.

Great Calculation draws from—or even is a partial translation of—Fibonacci's treatise is grounded on no evidence, and is in fact never seriously argued[131]. *Anonymus* 1252 shows no lexical loans from Italian; the form of the Hindu-Indian numerals does not indicate anything—for instance, in redacting his *Great Calculation*, Planudes shifts from the Western form of *Anonymus* 1252 to the Eastern form. As for *Anonymus* B, Mesarites' passage mentioned above shows that finger-notation was well-established in Byzantine schools before the Latin conquest.

As for the purposes of *Anonymi* B and 1252, one must bear in mind that they are literary products belonging to the scientific type "computational primer", a genre that was well-established since Late Antiquity; and in fact, neither Rhabdas nor Planudes had any problems in transforming them into overtly literary products. In particular, neither of the original treatises can be assumed to have any connection with the actual teaching of elementary operations in Byzantine schools—of course, no one would teach how to calculate square roots of non-square numbers at a primary school level[132]. If the issue seems open to question in the case of *Anonymus* B, just recall that this treatise does not explain how to actually perform the operations, nor does it provide any instruction for the use of the tables.

It is natural to come to doubt the authenticity of Rhabdas' *Letter to Tzavoukhes*, too. As we have seen, this treatise is almost complementary to the *First Letter*, while being much longer than it, and is composite in character. The problem of finding its sources, if any exist, is thus more difficult than in the case of the *First Letter*: the sectional nature of the *Letter to Tzavoukhes* most plausibly requires a plurality of sources; some of the independent sections are very short, thus making the identification of possible sources not easy[133]; the *Rechenbuch* closing the *Letter to Tzavoukhes* belongs to a kind of text that escape standard philological methods for establishing filiations[134]. And in fact, some (but only some) of the problems in this *Rechenbuch* coincide, as we have seen in n. 23, with problems in an anonymous *Rechenbuch* transcribed earlier than 1303. Further investigations and some plain good luck (see note added in proof on p. 37) will possibly enable us to assess better the issue of the sources of the *Letter to Tzavoukhes*.

CAN WE SPEAK OF "PLAGIARISM" AMONG BYZANTINE MATHEMATICAL AND NATURAL SCIENTIFIC WRITERS?

The above discussion "naturally" leads us to a thorny issue: Byzantine scientific production (and not only this) suffers from a diffusion of what *we* would call "plagiarism" among "colleagues": this

[131] A similar claim concerning the *Rechenbuch* he publishes is suggested but not argued in Vogel, Rechenbuch 154–160 and the all-inclusive table there attached, nor is it the one concerning *Anonymus* 1252 in Allard, Premier 60–64. Again, and contrary to the author's implicit contention, the analysis in the commentary of Allard, Planude, cannot prove anything about the relationships between Fibonacci's and Planudes' treatises. For a better-balanced assessment concerning *Rechenbücher*, see J. Høyrup, Fibonacci – Protagonist or Witness? Who Taught Catholic Christian Europe about Mediterranean Commercial Arithmetic? *Journal of Transcultural Medieval Studies* 1 (2014) 219–247: 236–238, who sees it as more likely a partial borrowing in the opposite direction, namely, "that the Italian and Iberian way to formulate alloying problems had its roots in a *Byzantine* money-dealers environment" (ibid., 238, emphasis in the original). Recall that Fibonacci claims three times that the problem at issue was proposed by a *magister constantinopolitanus* (*Liber abbaci*, ed. B. Boncompagni, Scritti di Leonardo Pisano, II. Roma 1857: 188, 190, 249). This is in fact the sole basis supporting the claim that Fibonacci was present in Constantinople at the end of 12th century.

[132] The fact that *Anonymus* B refrains from doing this, claiming that it is a difficult task even with the help of a teacher, seems to us to corroborate our point rather than to disprove it.

[133] A first attempt at identifying the source of the Easter Computus has been unsuccessful.

[134] The point is that these collections of disparate arithmetical problems can be assembled and de-assembled very easily, and that any such problem is conducive to (major) variant readings.

amounts to putting one's own name on a writing for the most part (or completely) composed by some other Byzantine author[135]. Cases in point include the following[136]:

- Maximos Planudes appropriated *Anonymus* 1252, as just seen.
- Rhabdas appropriated an entire treatise in his *First Letter*, as shown in the previous parts of this article.
- Chapters 8–10 of Book II of Theodoros Metochites' *Abridged Astronomical Elements* deal with compounded ratios and removal of a ratio from a ratio[137]. This is nothing but a plagiarism, with enormous amplifications (Metochites' fondness of περιβολή is legendary), of what we read in §§ 3–4 of the astronomical section of Pachymeres' *Quadrivium*.
- A part of an encyclopaedia redacted by the early 14th-century compiler Joseph Rhakendytes, based in Thessaloniki and in correspondence with Metochites, Moschopoulos, and Gregoras, is a mere plagiarism, with slight modifications, of the *Quadrivium* known as *Anonymus Heiberg*[138]. The plagiarism is so exact that Heiberg, in his edition, and P. Moore, who lists ninety-five manuscripts of the same work[139], include as witnesses of it a number of manuscripts which in fact contain Joseph's encyclopaedia.
- Argyros appropriated the method of Easter Computus expounded in Rhabdas' *Letter to Tzavoukhes* and wrote a treatise on the astrolabe that draws abundantly from Gregoras'[140].
- Theodoros Meliteniotes (d. 8/3/1393) presents as Book III of his *Three Books on Astronomy* an introduction to Persian astronomical tables almost certainly written by someone else[141].
- An anonymous computational primer to the *Almagest* is by and large made of extracts from the logistic portion of Pachymeres' *Quadrivium* and from Metochites' and Meliteniotes' primers[142].
- John Chortasmenos (d. 1431) wrote a tract on compounded ratios and removal of a ratio from a ratio[143] that is nothing but a verbatim plagiarism of a part of Book V of Barlaam's *Logistic*, in particular propositions 18–23.

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135 It goes without saying that Byzantine writers systematically plundered ancient Greek authors; we thus exclude from our list "quotations" such as those of the beginning of Diophantos' *Arithmetica* prefacing both of Rhabdas' *Letters*. Of course, we also exclude the *retractationes* and the borrowings from ancient writings used to compose Byzantine *Quadrivia*, since these are expressly conceived of as compilations.

136 All examples come from the Nicaean and the Palaiologan periods simply because almost nothing substantial has remained of the preceding scientific production (if anything substantial was ever produced).

137 We read them in Vat. gr. 181 (*Diktyon* 66812), ff. 26r–35r. We did not check other portions of Metochites' treatise. He died on March 13, 1332.

138 See U. Criscuolo, Note sull'«enciclopedia» del filosofo Giuseppe. *Byzantion* 44 (1974) 255–281. Ed. of *Anonymus Heiberg* in J. L. Heiberg, Anonymi Logica et Quadrivium (*Det Kongelige Danske Videnskabernes Selskabs, Historisk-filologiske Meddelelser* 15,1). København 1929.

139 P. Moore, Iter Psellianum. A detailed list of manuscript sources for all works attributed to Michael Psellos, including a comprehensive bibliography (*Subsidia Mediaevalia* 26). Toronto 2005, PHI.170. *Anonymus Heiberg* also circulated ascribed to Michael Psellos (b. 1018).

140 See Schissel, Osterrechnung, and A. Delatte, Anecdota Atheniensia et alia. Tome II. Textes grecs relatifs à l'histoire des sciences. Liège–Paris 1939, 193. Among other things, Argyros recycled almost verbatim the beginning of the preface of Gregoras' improved version of his own treatise; both writings can be read in autograph transcriptions, Gregoras' revision of his own tract in Vat. gr. 1087 (*Diktyon* 67718), ff. 313v–320v (with short breaks penned by other copyists: M. Menchelli, Struttura e mani del Vat. gr. 1087 [con osservazioni paleografiche sul copista C e il Marc. gr. Z. 330], in: Antiche stelle a Bisanzio. Il codice Vaticano greco 1087, ed. F. Guidetti – A. Santoni. Pisa 2013, 17–56: 38–40), Argyros' in Marc. gr. Z. 323, ff. 394r–400r.

141 For Meliteniotes' "sources" in other portions of his treatise, see the edition of Books I and II in R. Leurquin (ed.), Théodore Méliténiote, Tribiblos Astronomique. Livre I; Livre II (*Corpus des Astronomes Byzantins* 4–6). Amsterdam 1990–93, I 328–334, 406–412; II 877–883.

142 Ed. J. Moll, Étude sur un traité anonyme d'initiation à l'Almageste. I–II. Mémoire de licence. Louvain 1965.

143 We read it in the autograph Wien, Österreichische Nationalbibliothek, suppl. gr. 75 (*Diktyon* 71538), ff. 234r–256v, a copy of which is Ambros. C 263 inf. (*Diktyon* 42502), ff. 195r–212r (16th century).

One must say that the *extent* of the phenomenon, not its mere existence, is bewildering: ancient Greek mathematicians and commentators on mathematical matters not infrequently "forget" to indicate their sources; suffice to recall the striking similarity of some solutions to the problem of duplication of the cube[144], or the relations between Pappus' and Theon's commentaries on Ptolemy's *Almagest*. Maybe the diffusion of "plagiarism" is just another facet of Byzantine encyclopaedism, or compilatory habit, a consequence of culturally embedded strategies of authorial composition; think of Michael Psellos' wide-ranging literary output, to a large extent made of compilations of ancient sources.

Still, one must also take into account that we can assume that we have a fairly complete documentary record of Byzantine science (this is not the case for Ancient Greek science, and incompleteness cannot but reduce the phenomenon), and that Byzantine science was produced during a very limited period, more or less in one and the same place, by people belonging to a very restricted elite and therefore acquainted with each other. This acquaintance was frequently strengthened by a master-pupil relationship: in this case, appropriation is a form of faithfulness to the masters, authorial appropriation being just a form of intellectual appropriation. Another point to be considered in this respect is that we cannot assume that something written in, for example, 14th-century Constantinople, even redacted, even endowed with a preface in due form—but intended for a very limited readership—was necessarily felt as "published" and therefore to be connected with a well-defined author.

Clearly, the intellectual world of the European Middle Ages was not exempt from concerns over questions of authorship, authority, and authenticity and their significance in the realm of the literary. While the analysis of authorial representations inevitably raises questions of self- and personhood, the fragility and instability characteristic of a tradition of textual transmission from before the introduction of the printing press in Europe bring a different set of concerns, namely with authorial attribution, recycling, and anonymization of texts written by others. John Tzetzes famously complained about a student of his who planned to publish a detailed record of Tzetzes's lectures on the *Iliad* as a commentary under his own name[145]. While Tzetzes's example may be representative of the concerns of the professional intellectual in Komnenian Byzantium, whether Palaiologan authors of mathematical works experienced a similar anxiety concerning the practices of recycling and appropriation of their texts is a question that goes beyond the limits of the present study[146].

Note added in proof to page 35. Rhabdas' procedure in his Computus coincides with the one in Blastares' Σύνταγμα, 418–419 Rhalle – Potle. Reading primary sources is more effective than hoping to get a lucky break.

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144 Best account in W. R. Knorr, Textual Studies in Ancient and Medieval Geometry. Boston – Basel – Berlin 1989, 11–153. Read also Porphyry's complaint about Ptolemy's systematic plagiarisms in his *Harmonics*: I. Düring (ed.), Porphyrios Kommentar zur Harmonielehre des Ptolemaios. Göteborg 1932, 5.7–16.

145 On Tzetzes and discussions of authorship, see E. Cullhed, The Blind Bard and 'I': Homeric Biography and Authorial Personas in the Twelfth Century. *BMGS* 38:1 (2014) 49–67, in particular 61–67. For a wider discussion of authorship in Byzantine literature, applying various theoretical approaches and including further bibliography, see ed. A. Pizzone, The Author in Middle Byzantine Literature. Modes, Functions, and Identities. Berlin 2014.

146 Read, however, the ease with which Planudes treats the issue in his letter to Bekkos: τὸ δὲ πλέον ἑκάστης ἡμέρας, ἐξ οὗ τὴν βίβλον ἣν ἴστε παρ' ὑμῶν ἐχρησάμην, ὁ κατ' Ἰνδοὺς ἀριθμὸς δαπανᾷ καὶ θεοῦ διδόντος ἤδη τὸ πᾶν ἤνυσται. καί με οὐδὲν διέδρα τῶν ἐν αὐτῷ, πλὴν καὶ ταῦτα προσθεῖναι τῇ γραφομένῃ μοι βούλομαι βίβλῳ· […] εἰ δὴ ταῦτα τῶν ὑμετέρων που βιβλίων ἐντέτακται ἢ καὶ ἄλλως ἔστιν ὑμᾶς εἰδέναι, εὐκταῖα ἂν ἐμοὶ δράσαιτε, εἰ γράψαντες πέμψαιτε "I was spending most of every day on the Indian reckoning, beginning when I had from you the book you know, and God willing I have just completed the whole of it. I did not depart in anything from what is in it, except that I also want to add the following items in the book I am about to write: […] Now, if these items turn out to be included somewhere in books of yours or if so happens that you know of them in other ways, you would do something highly desirable, if you wanted to send them to me in written form". The text is at *Epistulae*, no. 46, 80.8–18 ed. P. A. M. Leone, Maximi monachi Planudis epistulae (*Classical and Byzantine Monographs* 18). Amsterdam 1991.